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Sumar

1. Neagu M., Crstici B. - Asupra ecuaţiei funcţionale a lui M.Bertolino în distribuţii 1
2. Boja N., Dăianu D., Hatvany Cs. - Despre structura de algebră booleană a unei extensii primale de corpuri 7
3. Rendi D., Rendi B. - Multimorfisme de grupuri şi inversa generalizată 9
4. Neagu M., Burlacu E. - Asupra compunerii extensiilor directe ale distribuţiilor 11
5. Izvercean P.N., Popescu C. - Reprezentarea spaţiilor de funcţii pe algebre 15
6. Hatvany Cs. - Multipli stabile în algebre Post 23
7. Popuţa V. - Derivari generalizate pe perechi Jordan . . 27
8. Burlacu E., Matei St. - Asupra unor ecuaţii funcţionale Booleene 31
9. Matei St., Burlacu E. - Asupra unor metode de rezolvare a ecuaţiilor Booleene şi Pseudo-Booleene . . 33
10. Neamţu N.N. - Metoda Laplace-Dirichlet pentru determinarea sumei unei serii de puteri 35
11. Dăianu D. - Bioconvergenţa functorială 37
12. Radu Fl. - F.V - morfisme şi secţiuni în fibrate vectoriale (II) 40
13. Stavre P., Klepp F.C. - Spaţii SemiEinstein 43
14. Niţoiu R., Hatvany Cs. - Proprietăţi de separare în spaţii bitopologice 47
15. Stan I. - Asupra unui rezultat neliniar din interpolarea complexă pentru n spaţii Banach 51
16. Găvruţa P. - O caracterizare a funcţiei Gamma 54
17. Lipovan O. - Asupra unei integrale probiliste 57
18. Tudor Gh. - Studiu asupra repartiţiei zerourilor funcţiilor analitice 61
19. Kovacs A. - Asupra rezolvării unei ecuaţii integrale din hidrodinamică (II). 65

20. Topuzu E. - Spectrul caracteristic asociat unui sistem discret 69
21. Goleţ I. - Asupra convergenţei mulţimilor închise şi funcţiilor distanţă în spaţii metrice probabiliste 73
22. Rendi D., Rendi B. - Multimorfisme de inele 76
23. Neamţu D., Neamţu N. - Rezolvarea unei ecuaţii funcţionale şi o proprietate 79
24. De Sabata C., Crăciun P., Marcu C. - Studiul efectului de seră realizat cu o peliculă de ulei . . 81
25. Crăciun P.-Metoda de măsurare a activităţii pulberilor slab radioactive cu camera de ionizare . . 85
26. Borneas M. - Unele observaţii asupra microparticulelor 87
27. Cristea M. - Asupra funcţiei de răspuns dielectric a solidelor în cîmp electric variabil . . . 89
28. De Sabata C., Marcu C., Gangăl M., Boldan M. - Utilizarea unui model energetic în interpretarea comportării unor pelicule electrolitice cuplate de Ni-Co 93
29. De Sabata C., Baea R., Boldan M. - Cercetări experimentale privind eficienţa de depunere a aliajului electrolitic NiCoP 105
30. De Sabata C., Baea R., Boldan M. - Asupra stabilităţii proprietăţilor magnetice ale aliajului NiCoP faţă de tratamente sau şocuri termice 112
31. De Sabata C., Cristea M., Ilie I., Brezeanu M. - Proprietăţile magnetice ale unor oxizi micşti obţinuţi prin procedeul de "feritizare" pe cale umedă 117

LUCRARILE SEMINARULUI DE MATEMATICA SI FIZICA
AL
INSTITUTULUI POLITEHNIC "TRAIAN VUIA"TIMISOARA
NOVEMBER 1985

CONTENTS, INHALT, SOMMAIRE

1. Neagu M., Cristici B. - About the functional equation of M.Bertolino in distributions 1
2. Boja N., Dăianu D., Hatvany Cs., - On the Boolean algebra structure of a primale extent of fields 7
3. Rendi D., Rendi B. - Multimorphisms of group and the generalized invers 9
4. Neagu M., Burlacu E. - Sur la composition des extensions directes des distributions 11
5. Izvercean P.N., Popescu C. - Representation of spaces of functions on algebras 15
6. Hatvany Cs. - Stable sets in post algebras 23
7. Popuţa V. - Derivations generalisees sur paires de Jordan 27
8. Burlacu E., Matei St. - On some functional Boolean equations 31
9. Matei St., Burlacu E. - Über einige Lósungsmethoden der Boole'schen und Pseudo-Boole'schen gleichungen 33
10. Neamţu N.N. - The Laplace-Dirichlet method for determining the sum of Power series 35
11. Dăianu D. - Functional biconvergence 37
12. Radu Fl. - Les F.V - Morphismes et les sections dans les fibres vectoriels (II) 40
13. Stavre P., Klepp F.C. - Semi-Einstein space 43
14. Niţoiu R., Hatvany Cs. - Separation properties in bitopological spaces 47
15. Stan I. - On a non-linear complex interpolation result for n Banach spaces 51
16. Găvruţă P. - A characterizations of gamma function . 54
17. Lipovan O. - About a probabilistic integral 57
18. Tudor Gh. - L'etude sur la repartition des zero des fonctions analytiques (IV) 61
19. Kovacs A. - Über das Lösen einer Integralgleichung aus des Hydrodynamik (II) 65

20. Topuzu E. - The characteristic spectrum associated to a linear discrete system 69
21. Goleţ I. - On convergence of closed sets in a probabilistic metric space and probabilistic distance functions 73
22. Rendi D., Rendi B. - Multimorphisms of dings (II) . . 76
23. Neamţu D., Neamţu N. - The solving of a functional equation add a property of the hyperbole . 79
24. De Sabata C., Crăciun P., Marcu C. - Study of the hothous effect using thin oil layers 81
25. Crăciun P.-Methode de mesure de l'activité des materiaux faible raduiactives avec la chambre de ionisation 85
26. Borneas M. - Some observations on microparticles . . . 87
27. Cristea M. - On the dielectric response function of solid matter in variable electric field . 89
28. De Sabata C., Marcu C., Gangăl M., Boldan M. - Interpretation of the behaviour of coupled electrolytic Ni-Co films using an energetic model 93
29. De Sabata C., Baea R., Boldan M. - Recherches expérimentales sur l'efficience de galvanisation de l'alliage électrolytique NiCoP 105
30. De Sabata C., Baea R., Boldan M. - Etude sur la stabilité des propriétés magnétique de l'alliage NiCoP aux traitements et aux chocs thermiques 112
31. De Sabata C., Cristea M., Ilie I., Brezeanu M. - Magnetic properties of certain mixed oxides obtained by wet "Ferritization" 117

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al
Institutului Politehnic "Traian Vuia" Timişoara
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ABOUT THE FUNCTIONAL EQUATION OF M.BERTOLINO IN DISTRIBUTIONS

Neagu M., Crstici B.

In this paper we obtaine the general solution in distributions of M.Bertolino's equation, [1], [4]. We use the usual notations of the theory of distributions by L.Schwartz, [3].

Definition 1. Let be $\mathcal{T}^{x,y,z}_{y+z} \in \mathcal{L}(\mathcal{D}_{x,y,z}; \mathcal{D}_{x,y})$ the operator definite by

$$\mathcal{T}^{x,y,z}_{y+z}[\varphi] = \int_{\mathbb{R}} \varphi(x,y-z,z)dz = \int_{\mathbb{R}} \varphi(x,z,y-z)dz \quad (1)$$

for any $\varphi \in \mathcal{D}_{x,y,z}$, and let be $\tilde{\mathcal{T}}^{x,y,z}_{y+z} \in \mathcal{L}(\mathcal{D}'_{x,y}; \mathcal{D}'_{x,y,z})$ the transposed operator of this operator, definite by

$$\langle \tilde{\mathcal{T}}^{x,y,z}_{y+z}[T], \varphi \rangle = \langle T, \mathcal{T}^{x,y,z}_{y+z}[\varphi] \rangle \quad (2)$$

for any $T \in \mathcal{D}'_{x,y}$ and $\varphi \in \mathcal{D}_{x,y,z}$.

Analog, let be $\mathcal{T}^{y,z}_{y+z} \in \mathcal{L}(\mathcal{D}_{y,z}; \mathcal{D}_{y})$ the operator definite by

$$\mathcal{T}^{y,z}_{y+z}[\psi] = \int_{\mathbb{R}} \psi(y-z,z)dz = \int_{\mathbb{R}} \psi(z,y-z)dz \quad (3)$$

for any $\psi \in \mathcal{D}_{y,z}$ and let be $\tilde{\mathcal{T}}^{y,z}_{y+z} \in \mathcal{L}(\mathcal{D}'_{y}; \mathcal{D}'_{y,z})$ the transposed operator of this operator, definite by

$$\langle \tilde{\mathcal{T}}^{y,z}_{y+z}[U], \psi \rangle = \langle U, \mathcal{T}^{y,z}_{y+z}[\psi] \rangle \quad (4)$$

for any $U \in \mathcal{D}'_{y}$ and $\psi \in \mathcal{D}_{y,z}$.

Proposition 1. If $f \in \mathcal{T}^{loc}_{x,y}$, then $g = \tilde{\mathcal{T}}^{x,y,z}_{y+z}[f] \in \mathcal{T}^{loc}_{x,y,z}$

$$g(x,y,z) = f(x,y+z) \quad (5)$$

Analog, if $f \in \mathcal{T}^{loc}_{y}$, then $g = \tilde{\mathcal{T}}^{y,z}_{y+z}[f] \in \mathcal{T}^{loc}_{y,z}$ and

it is definite by the relation

$$g(y,z) = f(y+z) \tag{6}$$

Definition 2. Let be $\mathcal{J}^{x,y,z}_{y,z} \in \mathcal{L}(\mathcal{D}_{x,y,z}; \mathcal{D}_{x,y,z})$ the operator definite by

$$\mathcal{J}^{x,y,z}_{y,z}[\varphi] = \varphi(x,z,y) \tag{7}$$

for any $\varphi = \varphi(x,y,z) \in \mathcal{D}_{x,y,z}$ and let be $\tilde{\mathcal{J}}^{x,y,z}_{y,z} \in \mathcal{L}(\mathcal{D}'_{x,y,z}; \mathcal{D}'_{x,y,z})$ the transposed operator of this operator, definite by

$$\langle \tilde{\mathcal{J}}^{x,y,z}_{y,z}[W], \varphi \rangle = \langle W, \mathcal{J}^{x,y,z}_{y,z}[\varphi] \rangle \tag{8}$$

for any $W \in \mathcal{D}'_{x,y,z}$ and $\varphi \in \mathcal{D}_{x,y,z}$.

Analog, let be $\mathcal{J}^{y,z}_{y,z} \in \mathcal{L}(\mathcal{D}_{y,z}; \mathcal{D}_{y,z})$ the operator definite by

$$\mathcal{J}^{y,z}_{y,z}[\psi] = \psi(z,y) \tag{9}$$

for any $\psi = \psi(y,z) \in \mathcal{D}_{y,z}$ and let be $\tilde{\mathcal{J}}^{y,z}_{y,z} \in \mathcal{L}(\mathcal{D}'_{y,z}; \mathcal{D}'_{y,z})$ the transposed operator of this operator, definite by

$$\langle \tilde{\mathcal{J}}^{y,z}_{y,z}[T], \psi \rangle = \langle T, \mathcal{J}^{y,z}_{y,z}[\psi] \rangle \tag{10}$$

for any $T \in \mathcal{D}'_{y,z}$ and $\psi \in \mathcal{D}_{y,z}$.

Definition 3. If $U \in \mathcal{D}'_x$ and $V \in \mathcal{D}'_y$, then the distribution $\tilde{\mathcal{P}}_{x,y}[U,V] \in \mathcal{D}'_{x,y}$, definite by the relation

$$\langle \tilde{\mathcal{P}}_{x,y}[U,V], \psi \rangle = \langle U, \langle V, \psi \rangle \rangle \tag{11}$$

for any $\psi \in \mathcal{D}_{x,y}$, is called their direct product.

Proposition 2. We have the following relations

$$\tilde{\mathcal{J}}^{x,y,z}_{y,z} \tilde{\mathcal{J}}^{x,y,z}_{y+z}[T] = \tilde{\mathcal{J}}^{x,y,z}_{y+z}[T], \quad (\forall)\ T \in \mathcal{D}'_{x,y} \tag{12}$$

$$\tilde{\mathcal{J}}^{y,z}_{y,z} \tilde{\mathcal{J}}^{y,z}_{y+z}[U] = \tilde{\mathcal{J}}^{y,z}_{y+z}[U], \quad (\forall)\ U \in \mathcal{D}'_{y} \tag{13}$$

$$\tilde{\mathcal{J}}^{x,y}_{x,y} \tilde{\mathcal{P}}_{x,y}[U,V] = \tilde{\mathcal{P}}_{x,y}[V,U]\ ;\ (\forall)\ U \in \mathcal{D}'_x,\ (\forall)\ V \in \mathcal{D}'_y \tag{14}$$

Defirition 4. Let be $\mathcal{U}^{x,y,z}_{x} \in \mathcal{L}(\mathcal{D}_x; \mathcal{D}_{x,y,z})$ the operator definite by

$$\mathcal{U}^{x,y,z}_{x}[\varphi] = \int_{\mathbb{R}^2} \varphi(x,y,z)\,dy\,dz \tag{15}$$

for any $\varphi \in \mathcal{D}_{x,y,z}$ and let be $\tilde{\mathcal{U}}_x^{x,y,z} \in \mathcal{L}(\mathcal{D}'_x; \mathcal{D}'_{x,y,z})$ the transposed operator of this operator, definite by

$$\langle \tilde{\mathcal{U}}_x^{x,y,z}[T], \varphi \rangle = \langle T, \mathcal{U}_x^{x,y,z}[\varphi] \rangle \tag{16}$$

for any $T \in \mathcal{D}'_x$ and $\varphi \in \mathcal{D}_{x,y,z}$.

The distribution $\tilde{\mathcal{U}}_x^{x,y,z}[T] \in \mathcal{D}'_{x,y,z}$ is called direct extension in $\mathcal{D}'_{x,y,z}$ of the distribution $T \in \mathcal{D}'_x$.

Both other direct extensions and the properties of these, [2], will be used below.

Definition 5. If $T \in \mathcal{D}'_{x,y}$; $U \in \mathcal{D}'_y$; $V \in \mathcal{D}'_z$, then the equation

$$\tilde{\mathcal{T}}_{y+z}^{x,y,z}[T] = \tilde{\mathcal{U}}_{y,z}^{x,y,z}[\tilde{\mathcal{P}}_{y,z}[U,V]] + \tilde{\mathcal{U}}_{x,y}^{x,y,z}[T] \tag{17}$$

is called the generalized equation in the distributions Bertolino's equation, [1].

Proposition 3. If $f \in \mathcal{J}_{x,y}^{loc}$; $g \in \mathcal{J}_y^{loc}$ and $h \in \mathcal{J}_z^{loc}$, then the equation

$$\tilde{\mathcal{T}}_{y+z}^{x,y,z}[f] = \tilde{\mathcal{U}}_{y,z}^{x,y,z}[\tilde{\mathcal{P}}_{y,z}[g,h]] + \tilde{\mathcal{U}}_{x,y}^{x,y,z}[f] \tag{18}$$

can also be written

$$f(x,y+z) = g(y)h(z) + f(x,y) \tag{19}$$

which is the Bertolino's equation in local integrable functions.

Proposition 4. If $T \in \mathcal{D}'_{x,y}$; $U \in \mathcal{D}'_y$ and $V \in \mathcal{D}'_z$, then we have the following rules of derivations

$$D_2 \tilde{\mathcal{T}}_{y+z}^{y,z}[U] = \tilde{\mathcal{T}}_{y+z}^{y,z}[DU]\ ;\ D_3 \tilde{\mathcal{T}}_{y+z}^{y,z}[U] = \tilde{\mathcal{T}}_{y+z}^{y,z}[DU]$$

in $\mathcal{D}'_{y,z}$ and also in $\mathcal{D}'_{x,y,z}$:

$$D_1 \tilde{\mathcal{T}}_{y+z}^{x,y,z}[T] = \tilde{\mathcal{T}}_{y+z}^{x,y,z}[D_1 T]\ ;\ D_2 \tilde{\mathcal{T}}_{y+z}^{x,y,z}[T] = D_3 \tilde{\mathcal{T}}_{y+z}^{x,y,z}[T] = \tilde{\mathcal{T}}_{y+z}^{x,y,z}[D_2 T]$$

$$D_1 \tilde{\mathcal{U}}_{y,z}^{x,y,z}[\tilde{\mathcal{P}}_{y,z}[U,V]] = 0;\ D_2 \tilde{\mathcal{U}}_{y,z}^{x,y,z}[\tilde{\mathcal{P}}_{y,z}[U,V]] = \tilde{\mathcal{U}}_{y,z}^{x,y,z}[\tilde{\mathcal{P}}_{y,z}[DU,V]]\ ;$$

$$D_3 \tilde{\mathcal{U}}^{x,y,z}_{y,z}\left[\tilde{\mathcal{F}}_{y,z}\left[U,V\right]\right] = \tilde{\mathcal{U}}^{x,y,z}_{y,z}\left[\tilde{\mathcal{F}}_{y,z}\left[U,DV\right]\right]$$

$$D_1 \tilde{\mathcal{U}}^{x,y,z}_{x,y}\left[T\right] = \tilde{\mathcal{U}}^{x,y,z}_{x,y}\left[D_1 T\right] ; \quad D_2 \tilde{\mathcal{U}}^{x,y,z}_{x,y}\left[T\right] = \tilde{\mathcal{U}}^{x,y,z}_{x,y}\left[D_2 T\right] ;$$

$$D_3 \tilde{\mathcal{U}}^{x,y,z}_{x,y}\left[T\right] = 0$$

Lemma. If $T \in \mathcal{D}'_{x,y}$ is the solution of the equation (17), then T is define by the relation

$$T = \tilde{\mathcal{U}}^{x,y}_{x}\left[A\right] + \tilde{\mathcal{U}}^{x,y}_{y}\left[B\right] \tag{20}$$

where $A \in \mathcal{D}'_x$ and $B \in \mathcal{D}'_y$.

Proof. Using equation (17) the operators of derivation D_1 and D_3 we have

$$D_1 D_3 \tilde{\mathcal{T}}^{x,y,z}_{y+z}\left[T\right] = 0$$

Because

$$D_1 D_3 \tilde{\mathcal{T}}^{x,y,z}_{y+z}\left[T\right] = \tilde{\mathcal{T}}^{x,y,z}_{y+z}\left[D_1 D_2 T\right]$$

results $D_1 D_2 T = 0$, and we have (20).

Theorem. The general solution in distribution of the equation (17) can have one of the following forms:

$$T = \tilde{\mathcal{U}}^{x,y}_{x}\left[A\right] + \alpha\gamma\,\tilde{\mathcal{U}}^{x,y}_{y}\left[y\right]; \quad U(y) = \alpha \; ; \; V(z) = \gamma z \tag{21}$$

$$T = \tilde{\mathcal{U}}^{x,y}_{x}\left[A\right] + \alpha^2\beta\,\tilde{\mathcal{U}}^{x,y}_{y}\left[1-e^{\delta y}\right]; \; U(y) = \alpha e^{\delta y} ; \; V(z) = \alpha\beta\,(1-e^{\delta z}) \tag{22}$$

$$T = \tilde{\mathcal{U}}^{x,y}_{x}\left[A\right] ; \quad U = C; \quad V = 0 \tag{23}$$

$$T = \tilde{\mathcal{U}}^{x,y}_{x}\left[A\right] ; \quad U = 0; \quad V = D \tag{24}$$

where $A \in \mathcal{D}'_x$, $C \in \mathcal{D}'_y$, $D \in \mathcal{D}'_z$ are arbitrary distributions, and $\alpha, \beta, \gamma, \delta$ are real arbitrary constants.

Proof. If $T \in \mathcal{D}'_{x,y}$ verifies the equation (17), according to lemma, we have

$$T = \tilde{\mathcal{U}}^{x,y}_{x}\left[A\right] + \tilde{\mathcal{U}}^{x,y}_{y}\left[B\right]$$

where $A \in \mathcal{D}'_x$ and $B \in \mathcal{D}'_y$.

Substituting this form of the T in (17) fields $A \in \mathcal{D}'_x$ and $B \in \mathcal{D}'_y$ it must verifies the relation

$$\tilde{u}_x^{x,y,z}[A] + \tilde{T}_{y+z}^{x,y,z}\left[\tilde{u}_y^{x,y}[B]\right] = \tilde{u}_{y,z}^{x,y,z}\left[\tilde{P}_{y,z}[U,V]\right] + \tilde{u}_x^{x,y,z}[A] + \tilde{u}_y^{x,y,z}[B]$$

from where

$$\tilde{T}_{y+z}^{x,y,z}\left[\tilde{u}_y^{x,y}[B]\right] = \tilde{u}_{y,z}^{x,y,z}\left[\tilde{P}_{y,z}[U,V]\right] + \tilde{u}_y^{x,y,z}[B] \qquad (25)$$

Applying to the equation (25) the operator $\tilde{T}_{y,z}^{x,y,z} \in \mathcal{L}(\mathcal{D}'_{x,y,z}; \mathcal{D}'_{x,y,z})$ according to proposition 2, fields

$$\tilde{T}_{y+z}^{x,y,z}\left[\tilde{u}_y^{x,y}[B]\right] = \tilde{u}_{y,z}^{x,y,z}\left[\tilde{P}_{y,z}[V,U]\right] + \tilde{u}_z^{x,y,z}[B] \qquad (26)$$

from where, with (25) and (26), results

$$\tilde{u}_{y,z}^{x,y,z}\left[\tilde{P}_{y,z}[U,V]\right] + \tilde{u}_y^{x,y,z}[B] = \tilde{u}_{y,z}^{x,y,z}\left[\tilde{P}_{y,z}[V,U]\right] + \tilde{u}_z^{x,y,z}[B] \qquad (27)$$

Relation (27) it can be written

$$\tilde{u}_{y,z}^{x,y,z}\left[\tilde{P}_{y,z}[U,V] + \tilde{u}_y^{y,z}[B]\right] = \tilde{u}_{y,z}^{x,y,z}\left[\tilde{P}_{y,z}(V,U] + \tilde{u}_z^{y,z}[B]\right]$$

and we have, in $\mathcal{D}'_{y,z}$:

$$\tilde{T}_{y+z}^{y,z}[B] = \tilde{P}_{y,z}[U,V] + \tilde{u}_y^{y,z}[B] = \tilde{P}_{y,z}[V,U] + \tilde{u}_z^{y,z}[B] \qquad (28)$$

Avoiding the trivial solution $B \equiv 0$ in the (28), which might lead to the solutions (23) and (24), considering $\psi(y) \in \mathcal{D}_y$, arbitrary fixed, for which

$$\langle U, \psi \rangle = c_1; \quad \langle V, \psi \rangle = c_2; \quad \langle B, \psi \rangle = c_3; \quad \langle 1, \psi \rangle = c_4$$

fields (in $\mathcal{D}'_z$, e.g.)

$$B = -\frac{c_2}{c_4} U + \frac{c_1}{c_4} V + \frac{c_3}{c_4} \qquad (29)$$

Substituting B in (28) and using the operators of derivation D_2 and D_3, we have

$$\mathcal{P}_{y,z}\left[DU,DV\right] = \mathcal{P}_{y,z}\left[DV,DU\right] \tag{30}$$

from where

$$DV = c_5 DU$$

or

$$V = c_5 U + c_6 \tag{31}$$

where c_5 and c_6 are real constant.

Substituting, in the relation (28), fields $U \in \mathcal{D}'_y$ verifies a equation in the distributions of this form

$$\mathcal{T}^{y,z}_{y+z}\left[U\right] = c\ \mathcal{P}_{y,z}\left[U,U\right] \tag{32}$$

where c is a real constant.

Equation (32) is the Cauchy equation in the distributions and its solutions are only indefinite derivable functions and thus the solutions (21) and (22) are obtained,

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ASUPRA ECUATIEI FUNCTIONALE A LUI M.BERTOLINO IN DISTRIBUTII

Rezumat

In această lucrare se obţine soluţia generală în distribuţii a ecuaţiei lui M.Bertolino.

Seminarul de Matematică şi Fizică
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ON THE BOOLEAN ALGEBRA STRUCTURE OF A PRIMALE EXTENT OF FIELDS.

Boja N., Dăianu D., Hatvany Cs.

0. Introduction. Let K/k be a finite extension of fields. We will denote by $\hat{K}$ the set of all overfields of a K field, and by $\check{K}$ the set of its all subfields. The intersection $\check{K} \cap \hat{k}$ constitutes a nonempty family of fields which we called extent of fields defined by the K/k extension and will be denoted by $[k,K] = F$.

In [3] had been introduced the notion of primale extension, of the form $k(\theta_1,\ldots,\theta_s)$, where θ_j $(j \in \overline{1,s})$ are roots of the minimal polynomials $\varphi_j = t^{m_j} - p_j$, $(m_j \in Z;\ m_j \geqslant 2)$, with p_j - prime numbers. An extend defined by a primale extension will bedenoted by $[k,K]^p$, or F^p.

In the above extent $[k,K] = F$ we define the binary operations

$$J' \vee J'' = k(J' \cup J'') = k(J')(J''), \qquad J' \wedge J'' = J' \cap J'' \tag{1}$$

for every $J', J'' \in F$. Further, an order relation on F can by defined by

$$J_1 \leq J_2 \Longleftrightarrow J_1 \vee J_2 = J_2. \tag{2}$$

The pair $(F, \leq) \doteq \mathcal{F}$ forms a complete atomic lattice.

We called primale sublattice associated to the pimale extension F^p the sublattice $(PF^p, \leq) \doteq \mathcal{PF}^p$ of the lattice $(F^p, \leq) = \mathcal{F}^p$ formed by all intermediary fields of the F^p extent, which are primale extensions of the k field.

This lattice is distributive.

1. Theorem. The primale sublattice $\mathcal{PF}^p$ has a structure of Boolean algebra.

In the $\mathcal{PF}^p$ lattice the elements 0 = k and 1 = K, which are the first and the last element respectively, are mutually complemented. Being given an arbitrary element $x \in PF^p - \{0,1\}$

we shall determine $y \in PF^p$ so that

$$x \wedge y = 0, \quad x \vee y = 1. \tag{3}$$

If $J_{i_1} = k(\theta_{i_1})$, $(i_1 \in \overline{1,s})$ are the simple extensions of the k field, which form the atomic elements of the lattice $\mathcal{F}$, then for $x = J_{i_1}$, we have $y = J_{i_2 \ldots i_s} = k(\theta_{i_2}, \ldots, \theta_{i_s})$, where $1 \leq i_1, i_2, \ldots, i_s \leq s$ with either index different and ordered on an ascending line and $\overline{1,s} \subset \mathbb{N}$.

Further, for every element $x = J_{i_1 \ldots i_m}$, with $1 \leq m \leq s-1$ from PF^p the element $\bar{x} = J_{i_{m+1}, \ldots, i_s}$ represents the y complementary which verify the relations (3).

Indeed, using the definition (1) for the latticeal operations in F, we have

$$J_{i_1 \ldots i_m} \vee J_{i_{m+1} \ldots i_s} = k(J_{i_1 \ldots i_m} \cup J_{i_{m+1} \ldots i_s}) = J_{1 \ldots s} = K$$

and

$$J_{i_1 \ldots i_m} \wedge J_{i_{m+1} \ldots i_s} = J_{\emptyset} = k.$$

It follows taht the centre $\mathcal{C}(PF^p)$ of the $\mathcal{PF}^p$ lattice coincides with the set PF^p itself, i.e. the considered lattice is complemented.

Moreover, as the respective lattice is distributive, as mentioned above, and based on a known proposition ([1]), it results thatevery element x, chosen as above, admits an single complement $\bar{x}$, and $J_{i_{m+1}, \ldots, i_s}$ is the greatest element of the lattice whose latticeal product with $J_{i_1 \ldots i_m}$ is the element $0 = k$.

From the previous statements it results that PF^p is a Boolean algebra $(PF^p, (\vee, \wedge, \bar{\ }, k, K))$ with binary operations defined by the (1) and (2) relations and the unary operation of the complementate elements defined by

$$\begin{aligned} \bar{\ } : PF^p &\to PF^p \\ J_{i_1 \ldots i_m} &\mapsto \bar{J}_{i_1 \ldots i_m} \overset{\Delta}{=} J_{i_{m+1}, \ldots, i_s}. \end{aligned} \tag{4}$$

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DESPRE STRUCTURA DE ALGEBRA BOOLEANA A UNEI EXTENSII PRIMALE DE CORPURI

Rezumat

In lucrare se arată că o extensie primală de corpuri este o latice distributivă complementată, adică o algebră Boole şi, prin aceasta, o algebră Post de ordinul doi.

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MULTIMORPHISMS OF GROUP AND THE GENERALIZED INVERS

Rendi D., Rendi B.

Let $(G_1,\circ)$ and $(G_2,*)$ be arbitrary groups with $\theta_1 \in G_1$ and $\theta_2 \in G_2$ the neutrally elements. One denote by $\overline{x}$ the invers element of x.

Definition 1. [2]. The multifunction $h:G_1 \longrightarrow G_2$ is a multimorphism if and only if:

$1^0)$ $h(x_1) * h(x_2) \subseteq h(x_1 \circ x_2)$

$2^0)$ $\overline{h(x_1)} \subseteq h(\overline{x}_1)$.

for every $x_1, x_2 \in G_1$.

Remark. It results from Definition 1 that $h(x_1) * h(x_2) = h(x_1 \circ x_2)$ and $\overline{h(x_1)} = h(\overline{x}_1)$.

If $h:G_1 \longrightarrow G_2$ is a multimorphism then one are [2].

a) $h(\theta_1) \subseteq G_2$, $h(G_1) \subseteq G_2$ are subgroups

b) $h(\theta_1) \subseteq h(G_1)$ is a normal subgroup

c) for every $x \in G_1$ and every $y \in h(x)$ one are $h(x) = y * h(\theta_1) = h(\theta_1 x y)$

d) Ker $h = \{ x \mid \theta_2 \in h(x) \} \subseteq G_1$ is a normal subgroup of G_1.

e) if h is onto then there exist a morphism $\overline{h}:G_1 \longrightarrow G_2/h(\theta_1)$ such that $p \circ h = \overline{h}$, where $p:G_2 \longrightarrow G_2/h(\theta_1)$ is the projection map.

Proposition 1. If $h:G_1 \longrightarrow G_2$ is a multimorphism onto, then $h^{-1}:G_2 \longrightarrow G_1$, $h^{-1}(y) = \{ x \mid y \in h(x) \}$ is a multimorphism one are Ker $h = h^{-1}(\theta_2)$ and Ker $h^{-1} = h(\theta_1)$.

Proof. It is immediately that Ker $h = h^{-1}(\theta_2)$ and Ker $h^{-1} = h(\theta_1)$. Let $y_1, y_2 \in G_2$ be arbitrary elements. Then

$x_1 \in h^{-1}(y_1)$ and $x_2 \in h^{-1}(y_2)$ we have $h(x_1 \circ x_2) =$

$= h(x_1) * h(x_2) \ni y_1 * y_2$. Hence $x_1 \circ x_2 \in h^{-1}(y_1 * y_2)$, i.e.

$h^{-1}(y_1) \circ h^{-1}(y_2) \subseteq h^{-1}(y_1 * y_2)$. Finally we have

$$x \in \overline{h^{-1}(y)} \Leftrightarrow \bar{x} \in h^{-1}(y) \Leftrightarrow h(\bar{x}) \ni y \Leftrightarrow$$

$$\Leftrightarrow \bar{y} \in \overline{h(\bar{x})} = h(x) \Leftrightarrow x \in h^{-1}(\bar{y}).$$

i.e. $h^{-1}(y) = h^{-1}(\bar{y})$.

Consequence 1. If $h:G_1 \longrightarrow G_2$ is a multimorphism onto and $\operatorname{Ker} h = \{e_1\}$ then $h^{-1}:G_2 \longrightarrow G_1$ is a morphism.

Consequence 2. If $h:G_1 \longrightarrow G_2$ is a multimorphism onto, then $\tilde{h}:G_1/\operatorname{Ker} h \longrightarrow G_2/\operatorname{Ker} h^{-1}$ defined by $\tilde{h}([x]) = [y]$ is an isomorphism where $y \in h(x)$ is an arbitrary element.

Definition 2. The multimorphism $g:G_1 \longrightarrow G_2$ is called a selection morphism for multimorphism $h:G_1 \longrightarrow G_2$ is $g(x) \subseteq h(x)$ for every $x \in G_1$.

One denote by MS(h) the family of selection morphisms for multimorphism $h:G_1 \longrightarrow G_2$.

Definition 3. For a multimorphism onto $h:G_1 \longrightarrow G_2$ the family $MS(h^{-1})$ is called the family of inverses of family MS(h).

Remark. If $h:G_1 \longrightarrow G_2$ is a multimorphism such that $G_0 + h(G_1) = G_2$ is a decomposition of G_2, then we consider the multimorphism $h_0:G_1 \longrightarrow G_2$ defined by $h_0(x) = h(x) + G_0$. The family $MS(h_0^{-1})$ is the family of inverses of family MS(h). Particularly if h is an omomoephism, then $OMS(h_0^{-1})$ is the family of generalized inverses for h.

Proposition 2. For a subgroup $G \subset G_1$ we have $G_1 = G + H$ if and only if there exist a multimorphism onto $h:G \longrightarrow G_1$ such that $i \in MS(h)$, where $i:G \longrightarrow G_1$ is the inclusion map.

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MULTIMORFISME DE GRUPURI SI INVERSA GENERALIZATA

Rezumat

In lucrare se stabilesc unele proprietăţi pentru multimorfisme de group şi se tratează inversa generalizată a unui multimorfism de selecţie pentru un multimorfism. Se dă o condiţie necesară şi suficientă pentru descompunerea unui grup în sumă directă.

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SUR LA COMPOSITION DES EXTENSIONS DIRECTES DES DISTRIBUTIONS

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I. FENYO ([1], [2]) a défini des extensions pour les variables des distributions dans certains cas particuliers. Certaines equations fonctionnelles ont été résolues dans le domain des distributions.

L. HORMANDER [3] a défini des extensions pour les variables des distributions sur variétés, à l'aide de la transformé de Fourier des distributions.

Dans [5] ont été défini des extensions générales des distributions de R^n vers R^{n+m} $(n, m \geq 1)$, par une méthode naturelle, directe (sans utiliser la transformé de Fourier).

A la base des opérations directes avec des distributions il y a les extensions directes des distributions qui seront défini ici sur R^3 (R^2) (pour simplifier les notations, sans restreindre la généralité). On présentera des propriétés de ces extensions, qui seront utilisées, dans la suite, pour traiter en général des équations fonctionnelles en distributions.

On va utiliser des notations usuelles de la théorie des distributions d'après L.SCHWARTZ [7].

Définition 1. Soit $\mathcal{U}_x^{x,y} \in \mathcal{L}(\mathcal{D}_{x,y}; \mathcal{D}_x)$, réspéctivement $\mathcal{U}_y^{x,y} \in \mathcal{L}(\mathcal{D}_{x,y}; \mathcal{D}_y)$ les opérateurs définis par les relations :

$$\mathcal{U}_x^{x,y}[\varphi] = \int_R \varphi(x,y)dy \qquad (1)$$

$$\mathcal{U}_y^{x,y}[\varphi] = \int_R \varphi(x,y)dx \qquad (2)$$

et soit $\tilde{\mathcal{U}}_x^{x,y} \in \mathcal{L}(\mathcal{D}'_x; \mathcal{D}'_{x,y})$, réspéctivement $\tilde{\mathcal{U}}_y^{x,y} \in \mathcal{L}(\mathcal{D}'_y; \mathcal{D}'_{x,y})$ les adjoints de ces opérateurs, définis par les rélations

$$\langle \tilde{\mathcal{U}}_x^{x,y}[T], \varphi \rangle = \langle T, \mathcal{U}_x^{x,y}[\varphi] \rangle \; ; \; (\forall)\, T \in \mathcal{D}'_x \, , \varphi \in \mathcal{D}_{x,y} \qquad (3)$$

$$\langle \tilde{\mathcal{U}}_y^{x,y}[T], \varphi \rangle = \langle T, \mathcal{U}_y^{x,y}[\varphi] \rangle \; ; \; (\forall)\, T \in \mathcal{D}'_y \, , \varphi \in \mathcal{D}_{x,y} \qquad (4)$$

Définition 2. Soit $\mathcal{U}_x^{x,y,z} \in \mathcal{L}(\mathcal{D}_{x,y,z};\mathcal{D}_x)$; $\mathcal{U}_y^{x,y,z} \in \mathcal{L}(\mathcal{D}_{x,y,z};\mathcal{D}_y)$ et $\mathcal{U}_z^{x,y,z} \in \mathcal{L}(\mathcal{D}_{x,y,z};\mathcal{D}_z)$, les opérateurs définis par les relations :

$$\mathcal{U}_x^{x,y,z}(\varphi) = \int_{R^2} \varphi(x,y,z)\,dydz \tag{5}$$

$$\mathcal{U}_y^{x,y,z}(\varphi) = \int_{R^2} \varphi(x,y,z)\,dxdz \tag{6}$$

$$\mathcal{U}_z^{x,y,z} = \int_{R^2} \varphi(x,y,z)\,dxdy \tag{7}$$

pour tout $\varphi \in \mathcal{D}_{x,y,z}$ et soit $\tilde{\mathcal{U}}_x^{x,y,z} \in \mathcal{L}(\mathcal{D}'_x;\mathcal{D}'_{x,y,z})$, $\tilde{\mathcal{U}}_y^{x,y,z} \in \mathcal{L}(\mathcal{D}'_y;\mathcal{D}'_{x,y,z})$ et $\tilde{\mathcal{U}}_z^{x,y,z} \in \mathcal{L}(\mathcal{D}'_z;\mathcal{D}'_{x,y,z})$ les adjoints de ces opérateurs définis par les relations :

$$\langle \tilde{\mathcal{U}}_x^{x,y,z}[T], \varphi \rangle = \langle T, \mathcal{U}_x^{x,y,z}[\varphi] \rangle \; ; \; (\forall)\, T \in \mathcal{D}'_x, \varphi \in \mathcal{D}_{x,y,z} \tag{8}$$

$$\langle \tilde{\mathcal{U}}_y^{x,y,z}[T], \varphi \rangle = \langle T, \mathcal{U}_y^{x,y,z}[\varphi] \rangle \; ; \; (\forall)\, T \in \mathcal{D}'_y, \varphi \in \mathcal{D}_{x,y,z} \tag{9}$$

$$\langle \tilde{\mathcal{U}}_z^{x,y,z}[T], \varphi \rangle = \langle T, \mathcal{U}_z^{x,y,z}[\varphi] \rangle \; ; \; (\forall)\, T \in \mathcal{D}'_z, \varphi \in \mathcal{D}_{x,y,z} \tag{10}$$

Définition 3. Soit $\mathcal{U}_{x,y}^{x,y,z} \in \mathcal{L}(\mathcal{D}_{x,y,z};\mathcal{D}_{x,y})$; $\mathcal{U}_{x,z}^{x,y,z} \in \mathcal{L}(\mathcal{D}_{x,y,z};\mathcal{D}_{x,z})$ et $\mathcal{U}_{y,z}^{x,y,z} \in \mathcal{L}(\mathcal{D}_{x,y,z};\mathcal{D}_{y,z})$, les opérateurs définis par les relations :

$$\mathcal{U}_{x,y}^{x,y,z}[\varphi] = \int_R \varphi(x,y,z)\,dz \tag{11}$$

$$\mathcal{U}_{x,z}^{x,y,z}[\varphi] = \int_R \varphi(x,y,z)\,dy \tag{12}$$

$$\mathcal{U}_{yz}^{x,y,z}[\varphi] = \int_R \varphi(x,y,z)\,dx \tag{13}$$

pour tout $\varphi \in \mathcal{D}_{x,y,z}$ et soit $\tilde{\mathcal{U}}_{x,y}^{x,y,z} \in \mathcal{L}(\mathcal{D}'_{x,y};\mathcal{D}'_{x,y,z})$; $\tilde{\mathcal{U}}_{x,z}^{x,y,z} \in \mathcal{L}(\mathcal{D}'_{x,z};\mathcal{D}'_{x,y,z})$ et $\tilde{\mathcal{U}}_{y,z}^{x,y,z} \in \mathcal{L}(\mathcal{D}'_{y,z};\mathcal{D}'_{x,y,z})$ les adjoints de ces opérateurs définis par les relations :

$$\langle \tilde{\mathcal{U}}_{x,y}^{x,y,z}[T], \varphi \rangle = \langle T, \mathcal{U}_{x,y}^{x,y,z}[\varphi] \rangle \; ; \; (\forall)\, T \in \mathcal{D}'_{x,y}, \varphi \in \mathcal{D}_{x,y,z} \tag{14}$$

$$\langle \tilde{\mathcal{U}}_{x,z}^{x,y,z}[T], \varphi \rangle = \langle T, \mathcal{U}_{x,z}^{x,y,z}[\varphi] \rangle \; ; \; (\forall)\, T \in \mathcal{D}'_{x,z}, \varphi \in \mathcal{D}_{x,y,z} \tag{15}$$

$$\langle \tilde{\mathcal{U}}_{y,z}^{x,y,z}[T], \varphi \rangle = \langle T, \mathcal{U}_{y,z}^{x,y,z}[\varphi] \rangle \; ; \; (\forall)\, T \in \mathcal{D}'_{y,z}, \varphi \in \mathcal{D}_{x,y,z} \tag{16}$$

Proposition 1. Les règles de composition suivantes ont lieu:

$$\mathcal{U}_x^{x,y,z} = \mathcal{U}_x^{x,y}\mathcal{U}_{x,y}^{x,y,z} = \mathcal{U}_x^{x,z}\mathcal{U}_{x,z}^{x,y,z}$$

$$\mathcal{U}_y^{x,y,z} = \mathcal{U}_y^{x,y}\mathcal{U}_{x,y}^{x,y,z} = \mathcal{U}_y^{y,z}\mathcal{U}_{y,z}^{x,y,z}$$

$$\mathcal{U}_z^{x,y,z} = \mathcal{U}_z^{x,z}\,\mathcal{U}_{x,z}^{x,y,z} = \mathcal{U}_z^{y,z}\,\mathcal{U}_{y,z}^{x,y,z}$$

$$\tilde{\mathcal{U}}_x^{x,y,z} = \tilde{\mathcal{U}}_{x,y}^{x,y,z}\,\tilde{\mathcal{U}}_x^{x,y} = \tilde{\mathcal{U}}_{x,z}^{x,y,z}\,\tilde{\mathcal{U}}_x^{x,z}$$

$$\bar{\mathcal{U}}_y^{x,y,z} = \bar{\mathcal{U}}_{x,y}^{x,y,z}\,\bar{\mathcal{U}}_y^{x,y} = \bar{\mathcal{U}}_{y,z}^{x,y,z}\,\bar{\mathcal{U}}_y^{y,z}$$

$$\hat{\mathcal{U}}_z^{x,y,z} = \hat{\mathcal{U}}_{x,z}^{x,y,z}\,\hat{\mathcal{U}}_z^{x,z} = \hat{\mathcal{U}}_{y,z}^{x,y,z}\,\hat{\mathcal{U}}_z^{y,z}$$

Proposition 2. Les règles de dérivation suivantes ont lieu : (par exemple) :

$$D_1\tilde{\mathcal{U}}_x^{x,y}[T] = \tilde{\mathcal{U}}_x^{x,y}[DT]\ ;\quad D_2\mathcal{U}_x^{x,y}[T] = 0\ ;\quad (\forall)\ T \in \mathcal{D}'_x$$

$$D_1\tilde{\mathcal{U}}_y^{x,y}[T] = 0\ ;\quad D_2\tilde{\mathcal{U}}_y^{x,y}[T] = \tilde{\mathcal{U}}_y^{x,y}[DT]\ ;\quad (\forall)\ T \in \mathcal{D}'_y$$

$$D_1\tilde{\mathcal{U}}_x^{x,y,z}[T] = \tilde{\mathcal{U}}_x^{x,y,z}[DT],\ D_2\tilde{\mathcal{U}}_x^{x,y,z}[T] = D_3\tilde{\mathcal{U}}_x^{x,y,z}[T] = 0;\ (\forall) T \in \mathcal{D}'_x$$

$$D_1\tilde{\mathcal{U}}_{x,y}^{x,y,z}[T] = \tilde{\mathcal{U}}_{x,y}^{x,y,z}[D_1T];\ D_2\tilde{\mathcal{U}}_{x,y}^{x,y,z}[T] = \tilde{\mathcal{U}}_{x,y}^{x,y,z}[D_2T];\ D_3\tilde{\mathcal{U}}_{x,y}^{x,y,z}[T] = 0\ ;\ (\forall)\ T \in \mathcal{D}'_{x,y}.$$

Proposition 3. Les indentifications suivantes ont lieu (par exemple) :

$$\tilde{\mathcal{U}}_x^{x,y}[T] = \tilde{\mathcal{U}}_x^{x,y}[U] \Leftrightarrow T = U \text{ in } \mathcal{D}'_x$$

$$\tilde{\mathcal{U}}_y^{x,y}[T] = \tilde{\mathcal{U}}_y^{x,y}[U] \Leftrightarrow T = U \text{ in } \mathcal{D}'_y$$

$$\tilde{\mathcal{U}}_x^{x,y,z}[T] = \tilde{\mathcal{U}}_x^{x,y,z}[U] \Leftrightarrow T = U \text{ in } \mathcal{D}'_x$$

$$\tilde{\mathcal{U}}_{x,y}^{x,y,z}[T] = \tilde{\mathcal{U}}_{x,y}^{x,y,z}[U] \Leftrightarrow T = U \text{ in } \mathcal{D}'_{x,y}$$

Proposition 4. Si $f \in \mathcal{J}_x^{loc}$, $g \in \mathcal{J}_y^{loc}$, alors $u = \tilde{\mathcal{U}}_x^{x,y}[f] + \tilde{\mathcal{U}}_y^{x,y}[g] \in \mathcal{J}_{x,y}^{loc}$ et elle est définie par la relation :

$$u(x,y) = f(x) + g(y)\ ,\quad x \in \mathbb{R}\ ,\ y \in \mathbb{R}$$

et si, par exemple, $f \in \mathcal{J}_z^{loc}$, $g \in \mathcal{J}_{x,y}^{loc}$ et $h \in \mathcal{J}_{y,z}^{loc}$, alors $v = \tilde{\mathcal{U}}_x^{x,y,z}[f] + \tilde{\mathcal{U}}_{x,y}^{x,y,z}[g] + \tilde{\mathcal{U}}_{y,z}^{x,y,z}[h] \in \mathcal{J}_{x,y,z}^{loc}$ et elle est définie par la relation :

$$v(x,y,z) = f(x) + g(x,y) + h(y,z);\ x \in \mathbb{R},\ y \in \mathbb{R},\ z \in \mathbb{R}$$

Remarque 1. Des telles "sommes directes" précisées dans Proposition 4 on trouve dans beaucoup d'équations fonction-

Remarque 2. À la base de toutes opérations dirèctes avec des distribution se trouvent les extensions directes de celles-ci.

Remarque 3. Comme une reciproque des opérations directes on peut interpréter la réstriction des distributions ([4],[6]) qui e à la base, de même, les extensions directes des distributions. Par exemple, la distribution $S_{x,y} \in \mathcal{D}'_{x,y}$ s'appèlle la réstriction de la distribution $T_{x,y,z} \in \mathcal{D}'_{x,y,z}$ dans $z = z_0$ si la relation suivante e lieu (en $\mathcal{D}'_{x,y,z}$) :

$$\lim_{\lambda \to 0} \langle T_{x,y,z_0+\lambda z}, \varphi(x,y,z) \rangle = \langle \tilde{U}^{x,y,z}_{x,y}[S], \varphi \rangle$$

pour tout $\varphi \in \mathcal{D}_{x,y,z}$.

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ASUPRA COMPUNERII EXTENSIILOR DIRECTE ALE DISTRIBUTIILOR

Rezumat

Se prezintă unele proprietăţi ale extensiilor directe ale tribuţiilor definite pe R^3 (R^2) ce vor fi utilizate ulterior tratarea generală a unor ecuaţii funcţionale în distribuţii.

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REPRESENTATION OF SPACES OF FUNCTIONS ON ALGEBRAS

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In this note the (symbolic) representation of spaces of complex functions on algebras over $\mathbb{C}$ is studied and some known [1], [2] results are generalized.

1. Definitions and examples. Let $V \subset \{f: D \subsetneq \mathbb{C} \longrightarrow \mathbb{C}\}$ be a space of functions defined over the same open set $D \subset \mathbb{C}$ and U be unital algebra over $\mathbb{C}$. We suppose that the functions $f \in V$ are differentiable on D and $\dot{f} \in V$ for any $f \in V$.

Definition 1.1. If,

1^0 there exists an isomorphism of vector spaces $\Psi: V \longrightarrow U$

2^0 there exists a (fixed) element $d \in U$ such that $\Psi(\dot{f}) = d\,\Psi(f)$ for any $f \in V$;

then we say that the space V is representable on the algebra U.

Definition 1.2. The element $d \in U$ is called differential operator.

Theorem 1.1. If there exists the operator $d \in U$ satisfying condition 2^0, then it is unique.

Remark 1.1. If the space V is representable on the algebra U then $\Psi(f^{(n)}) = d^n \Psi(f)$ for any $f \in V$ and $n \in \mathbb{N}$.

In the following using the algebra $\mathbb{C}$ we will define two algebras over $\mathbb{C}$, very useful in our examples.

Remark 1.2. Let $A = \mathbb{C} \times \mathbb{C} = \{(z,u) \mid z,u \in \mathbb{C}\}$. Clearly, A is a vector space over $\mathbb{C}$ by mean of the usual operations. If $(z,u) \in A$, $(z',u') \in A$ then we define a multiplication on A by $(z,u) \wedge (z',u') = (zz' - uu', uz' + u'z)$. We see easily, that A is a unital algebra over $\mathbb{C}$ with unit $e_0 = (0,1)$. On the vector space $B = \mathbb{C} \times \mathbb{C}$ we define a multiplication by $(z,u) \vee (z',u') = (zz' + uu', uz' + u'z)$. Analogously B is a unital algebra over $\mathbb{C}$ with unit $e_0 = (1,0)$.

Example 1.1. Let $\omega \in \mathbb{C}$, $\omega \neq 0$. We consider the functions $f_1, f_2 : \mathbb{C} \longrightarrow \mathbb{C}$ given by $f_1(z) = \cos \omega z$, $f_2(z) = \sin \omega z$ and denote by V the vector space on $\mathbb{C}$ generated by the functions f_1, f_2. Any element $f \in V$ can be uniquely writen in the form $f(z) = a_1 \cos \omega z + a_2 \sin \omega z$. The map $\Psi : V \longrightarrow A$ given by $\Psi(f) = (a_1, a_2)$ is an isomorphism of vector spaces. If we take $d = (0, -\omega)$, we have $\Psi(\dot{f}) = d \cdot \Psi(f)$, i.e. the space V is (symbolic) representable on the algebra A. The function $f : \mathbb{C} \longrightarrow \mathbb{C}$ defined by $f(z) = a \sin(\omega z + \varphi)$ is represented (by mean of definition 1.1) by the element $(a_1, a_2) \in A$, where $a_1 = a \sin \varphi$, $a_2 = a \cos \varphi$.

Example 1.2. Let $p, \alpha \in \mathbb{C}$, $p \neq 0$ and $f_1, f_2 : \mathbb{C} \longrightarrow \mathbb{C}$ given by $f_1(z) = e^{\alpha z} \operatorname{sh} pz$, $f_2(z) = e^{\alpha z} \operatorname{ch} pz$. We denote by V the space of functions over $\mathbb{C}$ generated by the functions f_1, f_2 and let $f \in V$, $f = a_1 f_1 + a_2 f_2$. The map $\Psi : V \longrightarrow B$ defined by $\Psi(f) = (a_1, a_2)$ is an isomorphism of vector spaces. If we take $d = (\alpha, p)$, we have $\Psi(\dot{f}) = d \cdot \Psi(f)$ i.e. the functions $f \in V$ are representable by the elements of the algebra B. The function $f : \mathbb{C} \longrightarrow \mathbb{C}$ defined by $f(z) = a \operatorname{sh}(pz + \varphi)$ is (symbolic) represented by the element $(a_1, a_2) \in B$, where $a_1 = a \operatorname{ch} \varphi$, $a_2 = a \operatorname{sh} \varphi$.

2. Representation of spaces of functions on polynomial algebras.

Let U be the algebra generated by the polynomial $\varphi \in \mathbb{C}[\lambda]$, $\varphi = \lambda^{n+1}$, $n \in N$ and denote by V the space of functions over $\mathbb{C}$ generated by the functions $\{f_i\}_{i=0,n}$, $f_i : \mathbb{C} \longrightarrow \mathbb{C}$, defined by $f_i(z) = \frac{z^i}{i!}$, $i = \overline{0,n}$. Then we have

Theorem 2.1. The space V is (symbolic) representable on the algebra U.

Proof. The algebra U is generated by a polynomial of the form $\varphi = \lambda^{n+1}$, so there exists a basis $\{e^0, e, \ldots, e^n\}$ satisfying $e^{n+1} = \theta$ and is clear that U is an algebra with proper radical. Any element $f \in V$ has the form $f = a_0 f_0 + a_1 f_1 + \ldots + a_n f_n$, $a_i \in \mathbb{C}$, and the map $\Psi : V \longrightarrow U$ given by $\Psi(f) = a_0 e^n + a_1 e^{n-1} + \ldots + a_n e + a_n 1$ is a isomorphism of vector spaces. If we take $d \in U$, $d = e$ we have $\Psi(\dot{f}) = d \cdot \Psi(f)$ and the theorem is proved.

Let $\omega \in \mathbb{C}$, $\omega \neq 0$. We consider the functions $g_j : \mathbb{C} \longrightarrow \mathbb{C}$, $j = \overline{0,1}$ given by $g_0(z) = \sin \omega z$, $g_1(z) = \cos \omega z$ and $f_i : \mathbb{C} \longrightarrow \mathbb{C}$

by $f_i(z) = \frac{z^i}{i!}$, $i = \overline{0,n}$.

Theorem 2.2. Let $W = \{f_i\, g_j\}_{\substack{i=\overline{0,n} \\ j=\overline{0,1}}}$ the space of functions generated by the functions $f_i\, g_j$. The space W is (symbolic) representable on the algebra $A \otimes U$.

Proof. Clearly, the spaces W and $A \otimes U$ has the dimension $2n + 2$. Let $\{(1,0), (0,1)\}$ be a basis in A and $\{e^0, e, \ldots, e^n\}$ a basis in U. The map $\Psi: W \longrightarrow A \otimes U$ defined by:

$$\Psi(a_{00}f_0g_0 + a_{01}f_0g_1 + a_{10}f_1g_0 + a_{11}f_1g_1 + \ldots + a_{n0}f_ng_0 +$$

$$+ a_{n1}f_ng_1) = a_{00}(1,0)\otimes e^n + a_{01}(0,1) \otimes e^n + a_{10}(1,0)\otimes e^{n-1} +$$

$$+ a_{11}(0,1) \otimes e^{n-1} + \ldots + a_{n0}(1,0) \otimes e^0 + a_{n1}(0,1) \otimes e^0$$

is an isomorphism of vector spaces. If we take $d \in A \otimes U$, $d = (1,0) \otimes e + \omega(0,1) \otimes e^0$, we have $\Psi(\dot{f}) = d.\Psi(f)$ and the theorem is proved.

Example 2.1. Let $\omega \neq 0$. We consider the functions $f_i: \mathbb{C} \longrightarrow \mathbb{C}$, $i = \overline{1,4}$ given by $f_1(z) = z \sin \omega z$, $f_2(z) = z \cos \omega z$, $f_3(z) = \sin \omega z$, $f_4(z) = \cos \omega z$ and denote by V the space of functions generated by the functions f_i, $i = \overline{1,4}$. Let U be the algebra generated by the polynomial $\varphi = \lambda^2$. Any element $f \in V$ has the form $f = a_1f_1 + a_2f_2 + a_3f_3 + a_4f_4$, $a_i \in C$, $i = \overline{1,4}$. We denote by $A \otimes U$ the tensor product of the algebras A and U and let $\{(1,0) \otimes e^0, (0,1) \otimes e^0, (1,0) \otimes e, (0,1) \otimes e\}$ by a basis in $A \otimes U$, where $\{(1,0), (0,1)\}$ is a basis in A and $\{e^0, e\}$ is a basis in U.

The map $\Psi: V \longrightarrow A \otimes U$ given by

$$\Psi(f) = a_1(1,0)\otimes e^0 + a_2(0,1) \otimes e^0 + a_3(1,0) \otimes e + a_4(0,1)\otimes e$$

is an isomorphism of vector spaces. If we take $d \in A \otimes U$, $d = (1,0)\otimes e + \omega(0,1)\otimes e^0$ it results that $\Psi(\dot{f}) = d \cdot \Psi(f)$ for any $f \in V$. The function $f: \mathbb{C} \longrightarrow \mathbb{C}$ given by $f(z) = (Az + B)\sin(\omega z + \varphi)$, where $A, B, \omega, \varphi \in \mathbb{C}$ $A \neq 0$, $B \neq 0$, $\omega \neq 0$, $\varphi \neq 0$ is represented in the algebra $A \otimes U$ by the element $a_1(1,0) \otimes e^0 + a_2(0,1) \otimes e^0 + a_3(1,0) \otimes e + a_4(0,1) \otimes e$, where $a_1 = A \cos \varphi$, $a_2 = A \sin \varphi$, $a_3 = B \cos \varphi$, $a_4 = B \sin \varphi$.

Let H_φ be the algebra generated by the polynomial $\varphi \in \mathbb{C}[\lambda]$, $\varphi = \prod_{i=1}^{n} (\lambda - \lambda_i)$, $\lambda_i \in \mathbb{C}$, $\lambda_i \neq \lambda_j$ for $i \neq j$. We denote by V

EGYETEM KÖZPONTI KÖNYVTÁRA

the space of functions generated by the functions $f_i: \mathbb{C} \longrightarrow \mathbb{C}$, $i = \overline{1,n}$ given by $f_i(z) = e^{\lambda_i z}$.

We have the following

Theorem 2.3. The space V is representable on the algebra H_φ.

Proof. It is known that if the algebra H_φ is generated by the polynomial $\prod_{i=1}^{n}(\lambda - \lambda_i)$ then there exists an idempotent orthogonal basis $\{e_i\}_{i=\overline{1,n}}$, $e_i^2 = e_i$, $e_i e_j = \theta$, for $i \neq j$ and H_φ is the direct sum of n algebras H_i, $i = \overline{1,n}$ isomorphic to $\mathbb{C}$. The map $\psi: V \longrightarrow H_\varphi$ defined by $\psi(a_1 f_1 + a_2 f_2 + \ldots + a_n f_n) = a_1 e_1 + a_2 e_2 + \ldots + a_n e_n$ is an isomorphism of vector spaces. If we take $d \in H_\varphi$, $d = \lambda_1 e_1 + \lambda_2 e_2 + \ldots + \lambda_n e_n$ it results that $\psi(\dot{f}) = d \cdot \psi(f)$ and the theorem is proved.

Example 2.2. We consider the functions $f_i: \mathbb{C} \longrightarrow \mathbb{C}$, $i = \overline{1,2}$ given by $f_1(z) = ze^{\delta z}$, $f_2(z) = e^{\delta z}$, $\delta \neq 0$ and denote by V the space of functions generated by f_i. We denote by U the algebra generated by the polynomial $\varphi \in \mathbb{C}[\lambda]$, $\varphi = \lambda^2$ and by $\mathbb{C} \otimes U$ the tensor product of the algebras $\mathbb{C}$ and U. If $\{e_1'\}$, $\{e_1, e_2\}$ are basis in $\mathbb{C}$ and U respectively, then $\{e_1' \otimes e_1, e_1' \otimes e_2\}$ is a basis in $\mathbb{C} \otimes U$. The map $\psi: V \longrightarrow \mathbb{C} \otimes U$ given by $\psi(a_1 f_1 + a_2 f_2) = a_1 e_1' \otimes e_1 + a_2 e_1' \otimes e_2$ is an isomorphism of vector spaces. If we take $d \in \mathbb{C} \otimes U$, $d = \delta e_1' \otimes e_1 + e_1' \otimes e_2$, it results that $\psi(\dot{f}) = d \cdot \psi(f)$. The function $f: \mathbb{C} \longrightarrow \mathbb{C}$ given by $f(z) = (\alpha z + \beta) e^{\delta z}$, where $\alpha, \beta \in \mathbb{C}$, is represented in the algebra $\mathbb{C} \otimes U$ by the element $\alpha e_1' \otimes e_1 + \beta e_1' \otimes e_2$.

3. Representation of spaces of functions of many variables on algebras

Let $V \subset \{u: D \subset \mathbb{R}^n \longrightarrow \mathbb{R}\}$ be a vector space of functions over the field of real numbers, defined on the same open set $D \subseteq \mathbb{R}^n$ and A be a unital real algebra.

We suppose that the functions $u \in V$ admit on D first order partial derivatives by every variable x_i, $i = \overline{1,n}$ and that $\frac{\partial u}{\partial x_i} \in V$ for any $i = \overline{1,n}$.

Definition 3.1. If

1° there exists an isomorphism of vector spaces $\psi: V \longrightarrow A$

2° there exists n (fixed) elements $d_i \in A$, $i = \overline{1,n}$ such that $\psi(\frac{\partial u}{\partial x_i}) = d_i \cdot \psi(u)$ for any $u \in V$ and $i = \overline{1,n}$;

then we say that the space V is (symbolic) representable on the algebra A.

Definition 3.2. The elements $d_i \in A$, $i = \overline{1,n}$ a called derivation operators.

Theorem 3.1. If the space of functions V is represented (by mean df definition 3.1) on the algebra A, then for a given Ψ the derivation operators are unique.

Remark 3.1. If the space of functions V is representable (by mean of definition 3.1) on the algebra A, then

$$\Psi\left(\frac{\partial^k v}{\partial x_1^{s_1}\partial x_2^{s_2}\dots\partial x_n^{s_n}}\right) = d_1^{s_1}d_2^{s_2}\dots d_n^{s_n}.\Psi(u);\ s_1+s_2+\dots+s_n = k$$

Let $D \subset \mathbb{C}$ an open set, $V \subset \{f:D \longrightarrow \mathbb{C} \mid f \text{ holomorphic on } D\}$ and $W = \{u:E \subset \mathbb{R}^2 \longrightarrow \mathbb{R} \mid \exists\ f \in V \text{ so that } \mathrm{Re}\ f = u\}$.

We suppose that V is a vector space on $\mathbb{C}$ containing no non-zero constant functions and $\dot{f} \in V$ for every $f \in V$. Clearly, in thase conditions W is a vector space on $\mathbb{R}$.

Theorem 3.2. The space of functions V is representable (by mean of definition 1.1) on an algebra A over $\mathbb{C}$ if and only if the space of functions W is representable (by mean of definition 3.1) on an algebra A' over $\mathbb{R}$.

Proof. We suppose that V is representable (by mean of definition 1.1) on an algebra A over $\mathbb{C}$. Then there exists an isomorphism of vector spaces $\Psi:V \longrightarrow A$ and a fixed element $d \in A$ such that $\Psi(\dot{f}) = d\cdot\Psi(f)$ for any $f \in V$.

Let $u \in W$. By the definition of the space W there exists $f \in V$ such that $\mathrm{Re}\ f = u$. We define then $\Phi:W \longrightarrow A$ by $\Phi(u) = \Psi(f)$. If there exist $f_1, f_2 \in V$ such that $\mathrm{Re}\ f_1 = \mathrm{Re}\ f_2 = u$, then $f_1 - f_2 = c$ and by the fact that V is a vector space it results that $c \in V$. But by our hypothesis on V, it results that $c = 0$ and so $f_1 = f_2$. Hence the map Φ is well-defined. Clearly Φ is an isomorphism of vector spaces between W and A (considered as spaces over the field of real numbers). As $\dot{f} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$, where $f = u + iv$, from the definition of Φ it results that

$$\Phi\left(\frac{\partial u}{\partial x}\right) = \Psi(\dot{f}) = d\cdot\Psi(f) = d\cdot\Phi(u)$$

On the other hand $\dot{f} = \frac{1}{i}\left(\frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y}\right)$, where $i\dot{f} = \frac{\partial u}{\partial y} + i\frac{\partial v}{\partial y}$ and hence

$$\Phi\left(\frac{\partial u}{\partial y}\right) = \Psi(i\cdot\dot{f}) = i\,\Psi(\dot{f}) = i.d.\Psi(f) = i\,d.\Phi(u)$$

If we take A' = A (considered as algebra over the field of real numbers) and $d_1 = d$, $d_2 = id$, it results that W is representable on the algebra A' by mean of definition 3.1 (where $x_1 = x$, $y_1 = y$).

Conversely, we suppose that W is representable on the algebra A' by mean of definition 3.1. Then there exists an isomor-

phism $\varphi: W \longrightarrow A'$ and there exist the fixed elements $d_1, d_2 \in A'$ so that, for any $u \in W$ we have

$$\varphi\left(\frac{\partial u}{\partial x}\right) = d_1 \varphi(u), \quad \varphi\left(\frac{\partial u}{\partial y}\right) = d_2 \varphi(u)$$

Using the algebra A' we define a new algebra A (over $\mathbb{C}$) so that

$$A = A' \times A' = \{(a,b) \mid a,b \in A'\}, \quad (a_1,b_1)+(a_2,b_2) = (a_1+a_2, b_1+b_2)$$

$$(\alpha + i\beta)(a,b) = (\alpha a - \beta b, \alpha b + \beta a); \quad \alpha + i\beta \in \mathbb{C}$$

$$(a_1,b_1) * (a_2,b_2) = (a_1a_2 - b_1b_2, a_1b_2 + a_2b_1)$$

It can be verified immediately that A is an algebra over $\mathbb{C}$ by mean of the above defined operations. Now let $f \in V$. The function f admits the representation $f = u + iv$ with $u \in W$. As V is a vector space over $\mathbb{C}$, it results that $-if \in V$ and as $-if = v - iu$, it results $v \in W$ too. So we can define $\Psi: V \longrightarrow A$ by $\Psi(f) = (\varphi(u), \varphi(v))$. It can be immediately verified that Ψ is an isomorphism of vector spaces over $\mathbb{C}$. Moreover,

$$\Psi(\dot{f}) = \Psi\left(\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\right) = \left(\varphi\left(\frac{\partial u}{\partial x}\right), \varphi\left(\frac{\partial v}{\partial x}\right)\right) =$$

$$= (d_1 \varphi(u), d_1 \varphi(v)) = (d_1,0) * (\varphi(u), \varphi(v)) =$$

$$= (d_1,0) * \Psi(f).$$

Taking $d = (d_1,0) \in A$, it results that V is representable by mean of definition 1.1 on the algebra A and the theorem is completely proved.

As every holomorphic function, thought as a function of a complex variable z, is equivalent (up to a constant) to a function of two real variables, the above theorem shows that the definition 3.1 is a natural extension of the representation of spaces of functions of one variable to spaces of function of many variables.

Let A be the algebra generated by the polinomial $\varphi \in \mathbb{R}[\lambda]$, $\varphi = \lambda^n - 1$, and $\{e^i\}_{i=\overline{0,n-1}}$ be a basis in A.

A function $f: D \subset A \longrightarrow A$ is defined by $f(x) = \sum_{i=0}^{n-1} u_i(x_0, x_1, \ldots, x_{n-1}) e^i$, where $u_i; E \subset \mathbb{R}^n \longrightarrow \mathbb{R}$, $i = \overline{0,n-1}$, and D is an open set. We denote by $\mathrm{Re}\, f = u_0$ and let $V \subset \{f: D \subset A \longrightarrow A \mid f$ holomorphis on $D\}$ respectively $W = \{u_0: E \longrightarrow \mathbb{R} \mid \exists f \in V$ so that $\mathrm{Re}\, f = u_0\}$. We suppose that the A-module V contains no non-zero constant functions. Clearly, in these conditions, W is a vector space. Let U be a unital algebra over A.

Definition 3.2. If

1^0 there exists an isomorphism of A-modules $\Psi: V \longrightarrow U$

2^0 there exists a (fixed) element $d \in U$ such that $\Psi(\dot{f}) = d\,\Psi(f)$ for any $f \in V$, then we say that the A-module V is representable on the A-algebra U.

Theorem 3.3. The A-module V is representable (by mean of definition 3.2) on the A-algebra U if and only if W is representable (by mean of definition 3.1) on the algebra U' over $\mathbb{R}$.

Proof. We suppose that the A-module V is representable (by mean definition 3.2) on the A-algebra U. Then there exists an isomorphism of A-modules $\Psi: V \longrightarrow U$ and there exists a fixed element $d \in U$ such that $\Psi(\dot{f}) = d\,\Psi(f)$ for any $f \in V$. We define then $\varphi: W \longrightarrow U$ by $\varphi(u_0) = \Psi(f)$. Clearly, under the given conditions, φ is an isomorphism of vector spaces between W an U (considered over the field of real numbers). As

$$\dot{f} = \frac{\partial u_0}{\partial x_0} + e\,\frac{\partial u_1}{\partial x_0} + \ldots + e^{n-1}\,\frac{\partial u_{n-1}}{\partial x_0}$$

it results that $\varphi(\frac{\partial u}{\partial x_0}) = \Psi(\dot{f}) = d \cdot \Psi(f) = d\,\varphi(u_0)$. On the other hand, [6],

$$\dot{f} = \frac{1}{e^i}\,(\frac{\partial u_0}{\partial x_i} + e\,\frac{\partial u_1}{\partial x_i} + \ldots + e^{n-1}\,\frac{\partial u_{n-1}}{\partial x_i})$$

where

$$e^i \cdot \dot{f} = \frac{\partial u_0}{\partial x_i} + e\,\frac{\partial u_1}{\partial x_i} + \ldots + e^{n-1}\,\frac{\partial u_{n-1}}{\partial x_i}$$

$$\varphi(\frac{\partial u_0}{\partial x_i}) = \Psi(e^i \dot{f}) = e^i \Psi(\dot{f}) = e^i d\,\Psi(f) = e^i d\,\varphi(u_0),$$

$$i = \overline{0,n-1}\ .$$

Taking then U' = U (considered as algebra over the field of real numbers) and $d_0 = d$, $d_1 = ed, \ldots, d_{n-1} = e^{n-1}d$ it results that W is representable (by mean of definition 3.1) on the algebra U'.

Conversely, we suppose that W is representable (by mean of definition 3.1) on the algebra U'. Then there exists an isomorphism $\varphi: W \longrightarrow U'$ of vector spaces and there exist the elements $d_i \in U'$, $i = \overline{0,n-1}$ such that $\varphi(\frac{\partial u_0}{\partial x_i}) = d_i \cdot \varphi(u_0)$ for any $u_0 \in W$ and $i = \overline{0,n-1}$.

Using the real algebra U' we define a new algebra U (over A) by:

$$U = U' \times U' \times \ldots \times U' = \{(a_0, a_1, \ldots, a_{n-1})\ a_i \in U',\ i = \overline{0,n-1}\}.$$

$$(a_0, a_1, \ldots, a_{n-1}) + (b_0, b_1, \ldots, b_{n-1}) = (a_0 + b_0, a_1 + b_1, \ldots, a_{n-1} + b_{n-1})$$

$$(\alpha e^0 + \alpha_1 e^1 + \ldots + \alpha_{n-1} e^{n-1}) \cdot$$

$=(\alpha_0 a_0+\alpha_1 a_{n-1}+\ldots+\alpha_{n-1}a_1,\ \alpha_0 a_1+\alpha_1 a_0+\ldots+\alpha_{n-1}a_2,\ \alpha_0 a_{n-1}+$
$+\alpha_1 a_{n-2}+\ldots+\alpha_{n-1}a_0)$, for any $\alpha_0 e^0+\alpha_1 e^1+\ldots+\alpha_{n-1}e^{n-1}\in A$

$(a_0,a_1,\ldots,a_{n-1})\wedge(b_0,b_1,\ldots,b_{n-1}) =$

$= (a_0b_0 + a_1b_{n-1}+\ldots+ a_{n-1}b_1,\ldots,a_0b_{n-1}+a_1b_{n-2}+\ldots+a_{n-1}b_0)$

for any $(a_0,a_1,\ldots,a_{n-1})$, $(b_0,b_1,\ldots,b_{n-1})\in U$.

It can be easily verified that U is an algebra over A by mean of the above operations.

Now let $f\in V$. Then f admits the representation $f=u_0+u_1e+\ldots$ $\ldots+u_{n-1}e^{n-1}$ with $u_0\in W$. As V is an A-module, it can be easily verified that $u_i\in W$ for any $i=\overline{0,n-1}$. Hence we can define $\Psi:V\to U$ by $\Psi(f)=(\varphi(u_0),\varphi(u_1),\ldots,\varphi(u_{n-1}))$.

One verifies immediately that Ψ is an isomorphism of A-modules.

Moreover:

$$\Psi(\hat{f}) = \left(\frac{\partial u_0}{\partial x_0}+e\frac{\partial u_1}{\partial x_0}+\ldots+e^{n-1}\frac{\partial u_{n-1}}{\partial x_0}\right.=$$

$$=\left(\varphi\left(\frac{\partial u_0}{\partial x_0}\right),\varphi\left(\frac{\partial u_1}{\partial x_0}\right),\ldots,\varphi\left(\frac{\partial u_{n-1}}{\partial x_0}\right)\right)=$$

$$=(d_0\varphi(u_0),d_0\varphi(u_1),\ldots,d_0\varphi(u_{n-1}))=$$

$$=(d_0,0,\ldots,0)\wedge(\varphi(u_0),\varphi(u_1),\ldots,\varphi(u_{n-1}))=$$

$$=(d_0,0,\ldots,0)\wedge\Psi(f).$$

Taking $d=(d_0,0,\ldots,0)\in U$, it results that the A-module V is represented (by mean of definition 3.2) on the A-algebra U, and the theorem is completely proved.

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REPREZENTAREA SPATIILOR DE FUNCTII PE ALGEBRE

Rezumat

In lucrare se studiază reprezentarea unor spaţii de funcţii mai multe variabile pe algebre.

Seminarul de Matematică şi Fizică
al
Institutului Politehnic "Traian Vuia" Timişoara
Noiembrie 1985

STABLE SETS IN POST ALGEBRAS

Hatvany Cs.

Let $P = \langle B, \mathfrak{m} \rangle$ be a Post algebra, where B is the Boolean algebra of the complemented elements of P and $\mathfrak{m}$ is the chain $\{e_0 = 0 < e_1 \ \ldots < e_{n-1} = 1\}$. For an element $x \in P$ we write $x = \sum_{i=1}^{n-1} x_i e_i$ for the monotonic representation of x.

1. Stable sets. Let S be an arbitrary but nonempty subset of P. For every $k \in \{1,2,\ldots,n-1\}$ we introduce the following sets : $S_k := \{x_k \in B \ , \ x \in S\}$, $S^k := \{x \in P \ , \ x_k \in S\}$ and we denote $S \cap B$ by S_B. Now we are in position to give the following

Definition 1. A subset $S \neq \emptyset$ of P is : (a) k-absorbent if $S_k \subseteq S$; (b) k-saturated if $S^k \subseteq S$; (c) k-stable if $S_B = S_k$ and $S = S^k$.

Clearly, a subset is k-stable iff it is k-absorbent and k-saturated.

The following two lemmas can be easily verified.

Lemma 1. A subset $S \neq \emptyset$ of P is : (a) k-absorbent iff $x \in S$ implies $x_k \in S$; (b) k-saturated iff $x \in P$, $x_k \in S$ implies $x \in S$; (c) k-stable iff $x \in S$ is equivalent to $x_k \in S$.

The smallest k-absorbent (k-saturated, respectively k-stable) set containing a given nonempty set S is called the k-absorbent (k-saturated, respectively k-stable) covering of S and will be denoted by k-abs(S) (k-sat(S) , respectively k-stab(S)).

Lemma 2. For every nonempty subset S of P we have : (a) $k\text{-abs}(S) = S_k \cup S$; (b) $k\text{-sat}(S) = S \cup S^k$; (c) $k\text{-stab}(S) = k\text{-abs}(S) \cup k\text{-sat}(S) = k\text{-abs}(k\text{-sat}(S)) = k\text{-sat}(k\text{-abs}(S))$.

Now it is clear that S is k-absorbent (k-saturated, respectively k-stable) iff k-abs(S) = S (k-sat(S) = S , respectively k-stab(S) = S).

2. Stable ideals. The Post ideals of order k introduced in [2] and [5] are precisely our k-stable proper ideals. To avoid cumbersome formulations, we consider P itself as a k-stable ideal for every $k \in \{1,2,\ldots,n-1\}$.

Lemma 3. If S is a k-absorbent subset of P, then the ideal $(S]$ generated by S is k-absorbent.

Proof. Let $x \in (S]$. Then there exist $s_1,\dots,s_m \in S$ such that $x \leq \sum_{j=1}^{m} s_j$, hence $x_k \leq \sum_{j=1}^{m} (s_j)_k$. By the fact that S is k-absorbent, it results that $(s_j)_k \in S$ for every $j = 1,2,\dots,m$. Hence $x_k \in (S]$.

Lemma 4. If I is a k-absorbent ideal, then k-sat(I) is a k-stable ideal.

Proof. Let $x,y \in$ k-sat(I). If $x,y \in I$, then $x+y \in I \subseteq$ k-sat(I). If $x,y \in I^k$, then $x_k, y_k \in I$, hence $(x+y)_k \in I$ and so $x+y \in$ k-sat(I). If $x \in I$ and $y \in I^k$, then $x_k, y_k \in I$, hence again $x+y \in$ k-sat(I).

Let $y \leq x \in$ k-sat(I). If $x \in I$, then $y \in I \subseteq$ k-sat(I). If $x \in I^k$, then $x_k \in I$. From $y \leq x$ we get $y_k \leq x_k$, hence $y_k \in I$ or equivalently, $y \in$ k-sat(I).

By the k-stable ideal generated by a nonempty subset S of P we mean the smallest k-stable ideal containing S.

Proposition 1. The k-stable ideal generated by a nonempty subset S of P is k-sat((k-abs(S)]) $= \{x \in P \mid \exists\, s_1,\dots,s_m \in S : x_k \leq \sum_{j=1}^{m} (s_j)_k\}$.

Proof. By Lemma 3 and Lemma 4 k-sat((k-abs(S)]) is a k-stable ideal of P which contains S.

To prove the required equality, let T be the set on the right side of this equality. If $x \in T$, then there exist $s_1,\dots,s_m \in S$ such that $x_k \in (k\text{-abs}(S)]$, hence $x \in$ k-sat((k-abs(S)]). It results that $T \subseteq$ k-sat((k-abs(S)]).

We will show that T is saturated. Let $x \in P$ and $x_k \in T$. Then for some elements $s_1,\dots,s_m \in S$ we have $(x_k)_k = x_k = \sum_{j=1}^{m} (s_j)_k$ and that means that $x \in T$.

Now we show that $(k\text{-abs}(S)] \subseteq T$. Let $x \in (k\text{-abs}(S)] = (S_k \cup S]$. Then there exist $s_1,\dots,s_p,s_{p+1},\dots,s_m \in S$ such that $x \leq \sum_{j=1}^{p} (s_j)_k + \sum_{j=p+1}^{m} s_j$. But $x_k \leq \sum_{j=1}^{p} (s_j)_k$, hence $x \in T$.

From $(k\text{-abs}(S) \subseteq T$ and $T =$ k-sat(T) we get that k-sat((k-abs(S)]) $\subseteq T$. Hence the required equality is proved.

Finally, let I be a k-stable ideal including S. Then k-abs(I) $=$ $= I \supseteq$ k-abs(S), $(I] = I \supseteq (k\text{-abs}(S)]$ and k-sat(I) $= I \supseteq$ $\supseteq$ k-sat((k-abs(S)]) and this ends the proof.

Every post algebra carries a De Morgan algebra structure given by the involutory anti-endomorphism $\bar{}: P \longrightarrow P$ defined $\bar{x} := \sum_{i=1}^{n-1} (x_{n-i})' e_i$, where $'$ is the Boolean complementation of the complemented elements. For every $b \in B$ we have $\bar{b} = b'$, hence $\bar{x}_i = \overline{x_{n-i}}$, $i = 1,2,\dots,n-1$. (See for instance [1] or [3]). If S

a nonempty subset of P , the set $\{\bar{x} , x \in S\}$ will be denoted by $\bar{S}$.

Proposition 2.(a)For every k-absorbent (k-saturated,k-stable) proper ideal I of P , $\bar{I}$ is an (n-k)-absorbent ((n-k)-saturated, (n-k)-stable) proper filter of P; (b) For every k-absorbent (k-saturated , k-stable) prime ideal I of P , P-I is a k-saturated (k-absorbent ,k-stable) prime filter of P; (c) For every k-absorbent (k-saturated , k-stable) prime ideal I of P , P-$\bar{I}$ is an (n-k)-saturated ((n-k)-absorbent , (n-k)-stable) prime ideal of P.

The proof is routine.

3.Stable congruences.P^2 has a canonical Post algebra stucture given by $P^2 = \langle B^2 , \mathfrak{M} \rangle$,where $\mathfrak{M}$ is now the chain $\{(e_0,e_0) = (0,0) < (e_1,e_1) < \ldots < (e_{n-1},e_{n-1}) = (1,1)\}$.Every element $(x,y) \in P^2$ has the unique monotonic representation $(x,y) = \sum_{i=1}^{n-1} (x_i,y_i)(e_i,e_i)$ hence $(x,y)_i = (x_i,y_i)$, i = 1,2,...,n-1 .

Let $\theta \subset P^2$ be a proper congruence on P.If θ is a k-absorbent (k-saturated , k-stable) subset of P^2 , we will say that θ is a k-absorbent (k-saturated , k-stable) congruence on P.We consider that P^2 itself is a k-absorbent (k-saturated , k-stable) congruence on P for every $k \in \{1,2,\ldots,n-1\}$.

Proposition 3.If θ is a k-stable proper congruence on P and $i \neq j$, then $(e_i,e_j) \in \theta$ iff $i,j \in \{0,1,\ldots,k-1\}$ and $(e_i,e_j) \notin \theta$ iff $i \in \{k,\ldots,n-1\}$ or $j \in \{k,\ldots,n-1\}$.

Proof.Consider $(e_i,e_j) \in \theta$ .This is equivalent to $(e_i,e_j)_k \in \theta$. But $(e_p)_q \in \{0,1\}$ for $p,q \in \{0,1,\ldots,n-1\}$.Because $i \neq j$ and θ is proper , $(e_i,e_j)_k \in \theta$ is equivalent to $(e_i)_k = (e_j)_k = \ldots = (e_i)_{n-1} = (e_j)_{n-1} = 0$, hence to $i,j \in \{0,1,\ldots,k-1\}$.

Proposition 4.If θ is a k-absorbent congruence on P , then k-sat(θ) is a k-stable congruence on P.

Proof.For every $x \in P$ we have $(x,x) \in \theta \subseteq$ k-sat(θ) = k-stab(θ). If $(x,y) \in$ k-sat(θ) , then $(x,y)_k \in \theta$ hence $(y,x)_k \in \theta$ and so $(y,x) \in$ k-sat(θ).If $(x,y),(y,z) \in$ k-sat(θ) , then $(x,y)_k,(y,z)_k \in \theta$ hence $(x,z)_k \in \theta$, therefor $(x,z) \in$ k-sat(θ).It follows that k-sat(θ) is an equivalence relation on P.If $(x,y),(u,v) \in$ k-sat(θ), then $(x,y)_k,(u,v)_k \in \theta$ hence $(xu,yv)_k,(x+u,y+v)_k \in \theta$ and therefor $(xu,yv),(x+u,y+v) \in$ k-sat(θ).

In [4] we investigate the the structure of the sets k-IDEAL(P) of all k-stable ideals of P and k-CONG(P) of all k-stable congruences on P and we will show that these sets are bounded distributive lattices isomorphic to the lattice IDEAL(B) of all ideals of the Boolean algebra B.

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MULTIMI STABILE IN ALGEBRE POST

Rezumat

In lucrare se introduce noţiunea de mulţime k-stabilă şi se studiază proprietăţi ale idealelor şi congruenţelor k-stabile în algebre Post.

Seminarul de Matematică şi Fizică
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DERIVATIONS GENERALISÉES SUR PAIRES DE JORDAN

Popuţa V.

Dans ce travail on donne une généralisation de la notion de dérivation classique sur paires de Jordan (voir [4]. I.3.1) et on étudie certaines propriétés.

Considerons une paire de Jordan linéaire $V = (V^+, V^-)$ sur le corps K de caracteristique différente de 2 et 3 et $f = (f_+, f_-)$ un endomorphisme de V, fixé.

Définition. Le couple $\Lambda = (\Lambda_+, \Lambda_-)$ d'applications K-linéaires $\Lambda_\sigma : V^\sigma \longrightarrow V^\sigma$, s'appelle dérivation sur V par rapport à f (ou suivant f) si

$$(1)\quad \Lambda_\sigma \{x,y,z\}_\sigma = \{\Lambda_\sigma(x), f_{-\sigma}(y), f_\sigma(z)\}_\sigma + \{f_\sigma(x), \Lambda_{-\sigma}(y), f_\sigma(z)\}_\sigma + \{f_\sigma(x), f_{-\sigma}(y), \Lambda_\sigma(z)\}_\sigma$$

quels que soient $x,z \in V^\sigma$, $y \in V^{-\sigma}$, $\sigma = \pm$.

Remarque 1. Si $f = (Id_{V^+}, Id_{V^-})$ et $\Lambda = (\Lambda_+, \Lambda_-)$ est une dérivation sur V par raport à f, alors (1) devient

$$(1')\quad \Lambda_\sigma \{x,y,z\}_\sigma = \{\Lambda_\sigma(x), y, z\}_\sigma + \{x, \Lambda_{-\sigma}(y), z\}_\sigma + \{x, y, \Lambda_\sigma(z)\}_\sigma$$

pour touts $x,z \in V^\sigma$, $y \in V^{-\sigma}$, $\sigma = \pm$, qui est la dérivation donnée par K.S.Watson (voir [4].I.3.1).

Remarque 2. L'ensemble $Der_f(V)$ des dérivations sur V par rapport a f, avec les opérations usuelles est un K-module.

Théorème 1. Λ est une dérivation sur V par rapport à f si et seulement si $f + \varepsilon\Lambda$ est un endomorphisme de $V_{K[\varepsilon]}$ (voir [3].0.3), où $K[\varepsilon]$ est l'algèbre de nombres duaux.

Demonstration. Le fait que $f + \varepsilon\Lambda = (f_+ + \varepsilon\Lambda_+, f_- + \varepsilon\Lambda_-)$ est un endomorphism de $V_{K[\varepsilon]}$ signifie que

$$(f_\sigma + \varepsilon\Lambda_\sigma)\{x_1 + \varepsilon x_2, y_1 + \varepsilon y_2, z_1 + \varepsilon z_2\}_\sigma =$$

$$=\left\{(f_\sigma+\varepsilon\Lambda_\sigma)(x_1+\varepsilon x_2),(f_{-\sigma}+\varepsilon\Lambda_{-\sigma})(y_1+\varepsilon y_2),(f_\sigma+\varepsilon\Lambda_\sigma)(z_1+\varepsilon z_2)\right\}_\sigma$$

et finalement, c'est equivalent à

$$\left\{f_\sigma(x_1),f_{-\sigma}(y_1),f_\sigma(z_1)\right\}_\sigma+\varepsilon\,\Lambda_\sigma\left\{x_1,y_1,z_1\right\}_\sigma =$$

$$=\left\{f_\sigma(x_1),f_{-\sigma}(y_1),f_\sigma(z_1)\right\}_\sigma+\varepsilon\left\{\Lambda_\sigma(x_1),f_{-\sigma}(y_1),f_\sigma(z_1)\right\}_\sigma+$$

$$+\varepsilon\left\{f_\sigma(x_1),\Lambda_{-\sigma}(y_1),f_\sigma(z_1)\right\}_\sigma+\varepsilon\left\{f_\sigma(x_1),f_{-\sigma}(y_1),\Lambda_\sigma(z_1)\right\}_\sigma$$

qui nous conduit à (1) et donc $\Lambda\in \mathrm{Der}_f(V)$ q.e.d.

Théorème 2. Λ est une dérivation sur V suivant f si et seulement si

$$\Lambda_\sigma(Q_\sigma(x)y)=\left\{\Lambda_\sigma(x),f_{-\sigma}(y),f_\sigma(x)\right\}_\sigma+Q_\sigma(f_\sigma(x))\Lambda_{-\sigma}(y) \tag{2}$$

quel que soit $x\in V^\sigma$, $y\in V^{-\sigma}$, $\sigma=\pm$, $Q_\sigma(x)$ étant la représentation quadratique de x.

Démonstration. Nécessité. Si Λ est une dérivation sur V suivant f, alors en prenant en (1) z = x, on obtient

$$\Lambda_\sigma\left\{x,y,x\right\}_\sigma=2\left\{\Lambda_\sigma(x),f_{-\sigma}(y),f_\sigma(x)\right\}_\sigma+\left\{f_\sigma(x),\Lambda_{-\sigma}(y),f_\sigma(x)\right\}_\sigma$$

d'où il resulte (voir [3].1.1.2)) la relation (2).

Suffisance. En supposant que (2) est vraie et en utilisant les relations (voir [3].1.1.(1))

$$\left\{x,y,z\right\}_\sigma=Q_\sigma(x,z)y,\ Q_\sigma(x,z)=Q_\sigma(x+z)-Q_\sigma(x)-Q_\sigma(z)\ ,$$

on obtient

$$\Lambda_\sigma\left\{x,y,z\right\}_\sigma=\Lambda_\sigma(Q_\sigma(x+z)y)-\Lambda_\sigma(Q_\sigma(x)y)-\Lambda_\sigma(Q_\sigma(z)y)=$$

$$=\left\{\Lambda_\sigma(x),f_{-\sigma}(y),f_\sigma(z)\right\}_\sigma+\left\{f_\sigma(x),\Lambda_{-\sigma}(y),f_\sigma(z)\right\}_\sigma+$$

$$+\left\{f_\sigma(x),f_{-\sigma}(y),\Lambda_\sigma(z)\right\}_\sigma\ ,\ x,z\in V^\sigma,\ y\in V^{-\sigma},$$

$$\sigma=\pm\ .\quad \text{q.e.d.}$$

Remarque 3. Si $f=\mathrm{Id}_V$, alors (2) devient

$$\Lambda_\sigma(Q_\sigma(x)y)=\left\{\Lambda_\sigma(x),y,x\right\}_\sigma+Q_\sigma(x)\Lambda_{-\sigma}(y), \tag{2'}$$

(voir [3].1.4).

Théorème 3. Si f est un élément central de End(V) et $\Lambda^1,\ \Lambda^2\in \mathrm{Der}_f(V)$, alors le crochet

$$\left[\Lambda^1,\Lambda^2\right]=\Lambda^1\circ\Lambda^2-\Lambda^2\circ\Lambda^1$$

est une dérivation sur V par rapport à f^2.

Remarque 4. Si f est un élément central de $\mathrm{End}(V)$ et $f^2 = f$, alors $\mathrm{Der}_f(V)$ est une algèbre de Lie (une sous-algèbre de $\mathrm{End}(V^+) \times \mathrm{End}(V^-)$).

Théorème 4. Le couple (f_+, f_-) est une dérivation sur V par rapport a $f = (f_+, f_-)$.

Théorème 5. Si $(a,b) \in V$, alors le couple

$$(3) \qquad \delta_f(a,b) = (f_+ \circ D_+(a,b),\ -f_- \circ D_-(b,a))$$

est une dérivation sur V suivant f, $D_\gamma(a,b)$ étant la différentielle définie par (a,b).

Démonstration. Si $x,z \in V^\gamma$, $y \in V^{-\gamma}$, alors (1) devient

$$(3.1) \quad (f_\gamma \circ D_\gamma(a,b))\{x,y,z\}_\gamma = \{f_\gamma \circ D_\gamma(a,b)(x), f_{-\gamma}(y), f_\gamma(z)\}_\gamma +$$
$$+ \{f_\gamma(x), -f_{-\gamma} \circ D_{-\gamma}(b,a)(y), f_\gamma(z)\}_\gamma + \{f_\gamma(x), f_{-\gamma}(y), f_\gamma \circ D_\gamma(a,b)(z)\}_\gamma$$

qui est équivalent à

$$(3.2) \quad f_\gamma\{a,b,\{x,y,z\}_\gamma\}_\gamma = \{f_\gamma\{a,b,x\}_\gamma, f_{-\gamma}(y), f_\gamma(z)\}_\gamma -$$
$$- \{f_\gamma(x), f_{-\gamma}\{b,a,y\}_{-\gamma}, f_\gamma(z)\}_\gamma +$$
$$+ \{f_\gamma(x), f_{-\gamma}(y), f_\gamma\{a,b,z\}_\gamma\}_\gamma .$$

Compte tenu du fait que $f \in \mathrm{End}(V)$ et de la relation (voir [1].I.§3)

$$\{a,b,\{x,y,z\}_\gamma\}_\gamma = \{\{a,b,x\}_\gamma, y, z\}_\gamma - \{x, \{b,a,y\}_{-\gamma}, z\}_\gamma +$$
$$+ \{x, y, \{a,b,z\}_\gamma\}_\gamma$$

il resulte que (3.2) est vraie. q.e.d.

Remarque 5. Si $f = \mathrm{Id}_V$, alors (3) devient

$$(3') \qquad \delta_{\mathrm{Id}_V}(a,b) = \delta(a,b) = (D_+(a,b),\ -D_-(b,a)),$$

(voir [3].3.11.(1)).

Définition. La dérivation $\delta_f(a,b)$ donnée par (3) s'appelle la dérivation intérieure sur V par rapport à f définie par le couple (a,b).

Théorème 6. Si $f^2 = f$ et f est un élement central de $\mathrm{End}(V)$, alors pour tout $\Lambda \in \mathrm{Der}_f(V)$ on a

$$(4) \qquad [\Lambda, \delta_f(a,b)] = \delta_f(\Lambda_+(a), b) + \delta_f(a, \Lambda_-(b))$$

Démonstration. Compte tenu du (3), la relation (4) est équivalente à

$$\Lambda_\gamma \circ f_\gamma \circ D_\gamma(a,b) - f_\gamma \circ D_\gamma(a,b) \circ \Lambda_\gamma = f_\gamma \circ D_\gamma(\Lambda_\gamma(a), b) + f_\gamma \circ D_\gamma(a, \Lambda_{-\gamma}(b))$$

ou, pour tout $x \in V^\gamma$,

$$\Lambda_\sigma\{f_\sigma(a),f_{-\sigma}(b),f_\sigma(x)\}_\sigma - f_\sigma\{a,b,\Lambda_\sigma(x)\}_\sigma = f_\sigma\{\Lambda_\sigma(a),b,x\}_\sigma + f_\sigma\{a,\Lambda_{-\sigma}(b),x\}_\sigma .$$

Finalement, on obtient:

$$\{\Lambda_\sigma(f_\sigma(a)),f^2_{-\sigma}(b),f^2_\sigma(x)\}_\sigma + \{f^2_\sigma(a),\Lambda_{-\sigma}(f_{-\sigma}(b)),f^2_\sigma(x)\}_\sigma +$$
$$+ \{f^2_\sigma(a),f^2_{-\sigma}(b),\Lambda_\sigma(f_\sigma(x))\}_\sigma - \{f_\sigma(a),f_{-\sigma}(b),f_\sigma(\Lambda_\sigma(x))\}_\sigma =$$
$$= \{f_\sigma(\Lambda_\sigma(a)),f_{-\sigma}(b),f_\sigma(x)\}_\sigma + \{f_\sigma(a),f_{-\sigma}(\Lambda_{-\sigma}(b)),f_\sigma(x)\}_\sigma ,$$
$$\sigma = \pm$$

qui est une relation vraie, parce que $f^2 = f$ q.e.d.

Remarque 5. La relation (4) est une généralisation de la relation (3) de (voir [3].3.11).

Théorème 7. Si f est un élément central de End(V), alors, pour tout $h \in Aut(V)$ on a

(5) $\quad h\,\delta_f(a,b)h^{-1} = \delta_f(h_+(a),h_-(b))$

Remarque 6. Si $f^2 = f$ et f est un élément central de End(V), alors l'enveloppe linéaire $I\,Der_f(V)$ des dérivations intérieures sur V par rapport à f, $\delta_f(a,b)$, $(a,b) \in V$ est un idéal dans l'algèbre de Lie $Der_f(V)$, stable par rapport aux automorphismes de V.

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DERIVARI GENERALIZATE PE PERECHI JORDAN

Rezumat

In lucrare se dă o generalizare a noţiunii de derivare pe perechi Jordan şi se studiază anumite proprietăţi.

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ON SOME FUNCTIONAL BOOLEAN EQUATIONS

Burlacu E. , Matei St.

Is known that a Boolean function $f : B \longrightarrow B$ (where B is a Boolean algebra) can be writen in the form $f(x) = ax \cup bx = (a+b)x + b$ where $a = f(1)$ and $b = f(0)$ are fixed elements from B.

In [1] is shown that a Boolean function can be writen in the form

$$f(x) = a_0.0 \cup a_1 x \cup a_2 x \cup a_3 = a_0.0 + a_1 x + a_2 x + a_3 \qquad (1)$$

where (a_0, a_1, a_2, a_3) is the orthonormal characteristic of f. Several properties of Boolean function can be expresed in terms of its orthonormal characteristic. Cf.R.A.Melter and S.Rudeanu [1] theorem 3 we have :

If $f = (a_0, a_1, a_2, a_3)$ then

(i) f is isotone iff $a_3 = 0$; (ii) f is antitone iff $a_1 = 0$; (iii) f is constant iff $a_1 = a_2 = 0$; (iv) f is injective iff $a_0 = a_2 = 0$; (v) f is simple iff $a_0 a_1 a_2 a_3 \in \{0, 1\}$.

The aim of this peper is to obtain the general solutions for various type of functional Boolean equations.

Proposition 1. The solutions of Boolean equations

$f(xy) = f(x)$	(2)		$f(x) = cx$ where $c = f(1) = a_1 \cup a_3$	(2')
$f(xy) = f(x)$	(3)	are	$f(x) = c$ where $c = f(1) = a_1 \cup a_3$	(3')
$f(x\,y) = f(x)$	(4)	respectively	$f(x) = c_1$ where $c_1 = f(0) = a_2 \cup a_3$	(4')
$f(x\,y) = f(x)\,y$	(5)		$f(x) = c_1$ " $c_1 = f(0) = a_2 \cup a_3$	(5')

The proof is obvious.

Note that equations of the same type are obtained if in the instead of conjunction or disjunction from 2-5 equations on put other binaire Boolean operations. Cauchy's basic equations

$$f(x \square y) = f(x) \square f(y) \qquad (6)$$

where $\square$ is one of the binaire operations of Boolean algebra.

The solution of this equation depend on the choised operation. We shall take into account same cases of equations.

Proposition 2. The solution of the functional equation

$$f(x \cup y) = f(x) \cup f(y) \qquad (7)$$

is the set of isotone functions.

Proof. If in (7) we put $y_1 = o$ we obtain $f(x) = f(x) \cup f(o)$ so $f(o) \leq f(x)$ $\forall x \in B$ then $y_1 = 1$ we obtain $f(1) = x(1) \cup f(x)$ that is $f(x) \leq f(1)$ $\forall x \in B$. From the last two inequality results $f(o) \leq f(1)$ that is f is isotone so $f(x) = a_1 x \cup a_3$ and $a_2 = 0$.

Obs. We obtain the same result in instead of operation $\square$ we put the conjunction.

Proposition 3. The functional Boolean equation

$$f(xy) = f(x) \cup f(y) \tag{8}$$

has as solution the set of antitone functions.

The proof is analogue to that of Proposition 2.

Proposition 4. The functional Boolean equation

$$f(f(x)) = f(x) \tag{9}$$

has as solution the set of isotone functions.

Proof. Let $f(x) = ax + b\bar{x}$. Replace $f(f(x)) = a\left[ax + b\bar{x}\right] + b\overline{\left[ax+b\bar{x}\right]} =$ $= ax + ab\bar{x} + b(a\bar{x} + b\bar{x} + 1) = ax + abx + ab + abx + bx + b + b =$ $= ax + bx + ab$; $f(f(x)) = f(x) \Rightarrow ax + bx + ab = ax + bx + b \Rightarrow$ $ab = b \Rightarrow a \supset b$. But $a \supset b \Rightarrow a = b \cup h$ so $f(x) = b \cup hx \Rightarrow a_2 =$ $= 0$ that means the function is isotone.

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ASUPRA UNOR ECUAŢII FUNCŢIONALE BOOLEENE

Rezumat

In lucrare sînt considerate cîteva ecuaţii funcţionale booleene şi se caută mulţimea soluţiilor. Se rezolvă ecuaţii de tip Cauchy şi se arată că soluţiile lor depind de operaţiile binare considerate.

Seminarul de Matematică şi Fizică
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ÜBER EINIGE LÖSUNGSMETHODEN DER BOOLE'SCHEN UND PSEUDO-BOOLE'SCHEN GLEICHUNGEN

Matei St., Burlacu E.

I. Die Auflösung der boole'schen Gleichungen mit Hilfe der ternären Matrizen.

Es sei die boole'schen Gleichung:

(1) $f(x_1,x_2,\dots,x_n) = 1;\quad f:\mathcal{B}_2^n \to \mathcal{B}_2;\quad \mathcal{B}_2 = \{0,1\}$

welche wir in der Gestalt:

(2) $f(X) = C_1(X) \oplus C_2(X) \oplus \dots \oplus C_m(X)$ schreiben.

Die Gestalt (2) kann in Gestalt einer ternären Matrix F mit den Elementen f_{ij}; $i = \overline{1,n}$; $j = \overline{1,m}$ welche die werte 1, 0, -, je nach dem als die Veranderliche x_i im Konjunktion C_j nicht-komplementiert, komplementiert order garnicht auftascht. Man ordnet die Entwicklung der Funktion f(X) in Bezung auf die Veränderliche x_i und man erhält:

(3) $f(X) = \overline{x}_i\, {}^{x_i}f^0 \cup x_i\, {}^{x_i}f^1$ worbei $\begin{cases} {}^{x_i}f^0 = f(Y,0) \\ {}^{x_i}f^1 = f(Y,1) \end{cases}$

die Koeffizienten der disjunktiven Entwicklung für

$Y = (x_1,\dots,x_{i-1},x_{i+1},\dots,x_n);\quad X = (Y,Y_i)$

woraus man die Beziehungen:

(4) $f(X) = x_i f_{x_i} \oplus {}^{x_i}f^0$ und

(5) $f(X) = \overline{x}_i f_{x_i} \oplus {}^{x_i}f^1$ wobei $f_{x_i} = {}^{x_i}f^0 \oplus {}^{x_i}f^1$ die

boole'sche Differenz der Funktion f(X) in Bezung auf die Veränderliche x_i. Die Koeffizienten der Entwicklung findet man leicht aus Matrix F. Die Funktionen ${}^{x_i}f^0$, ${}^{x_i}f^1$, f_{x_i} findet man mit Hilfe der Minoren Determinanten von F durch Untedrückung der Spalte des Ranges i und jener Zeilen die in der Spalte i bezichungsweise 1,0 order-

ent - halten. Unter Anwendung der Bezichungen 3, 4, 5 findet man auch die Lösungen der Gleichung (1) mit Hilfe der Lösungen der einfacheren Gleichungen:

(6) $\quad {}^{x_i}f^0 = 1 \; ; \quad {}^{x_i}f^1 = 1 \text{ oder } \quad f_{x_i} = 1.$

Satz 1. i) Ist a eine Wurzel der Gleichung ${}^{x_i}f^{\alpha} = 1$ dann ist (a,α) eine Wurzel der Gleichung $f(X) = 1$.

ii) ist b eine Wurzel der Gleichung $f_{x_i} = 1$ und β der Wert der Funktion ${}^{x_i}f^{\alpha}$ für $y = b$ dann ist $(b, \alpha \oplus \beta \oplus 1)$ eine Wurzel der Gleichung $f(X) = 1$.

iii) $f(X) \equiv 0$ dann und nur dann wenn ${}^{x_i}f^0 \equiv 0 \equiv {}^{x_i}f^1 \equiv f_{x_i}$

Der Beweis des Satzes erfolgt unter Anwendung der Beziehungen (2), (3), (4) und (5).

Betrachtung 1. Eine ähnliche Methode erhält man für die pseudo-boole'schen Gleichungen und Ungleichungen ($f: \mathcal{B}_2^n \to R$).

II. Die Auflösung der pseudo-boole'schen Gleichungen und Ungleichungen mit Hilfe einiger Auflösungen funktionen

Man betrachtet $f: S_p^n \to C$ wobei $S_p = \{0,1,\ldots,p-1\}$ und die pseudo-boole'sche Gleichung:

(7) $\quad f(x_1,x_2,\ldots,x_n) = \sum_{(\alpha_1,\ldots\alpha_n)} S_p^n \, a_{\alpha_1\ldots\alpha_n} x_1^{\alpha_1} x_2^{\alpha_2} \ldots x_n^{\alpha_n}$

Satz 2. Es sei $f(X) \varphi o$ eine mögliche Beziehung, wobei φ eine der Beziehungen: $\geq, \leq, >, <, =$. Die allgemeine Lösung dieser Beziehung ist durch die Gleichung:

(8) $X = A(q)(f(q), f(\alpha q), \ldots, f(\alpha^{p-1} q))$, $q \in S_p$, α_i Zyklus der Abbildungen von S_p in S_p, gegeben.

Die Allgemeine Lösung der pseudo-boole'schen Gleichung kann in der Gestalt

(9)
$$\begin{cases} x_n = x_n(q_n) \\ x_{n-1} = x_{n-1}(q_{n-1}, q_n) \\ \cdots\cdots\cdots\cdots\cdots \\ x_1 = x_1(q_1, q_2, \ldots, q_n) \end{cases}$$
geschrieben werden.

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ASUPRA UNOR METODE DE REZOLVARE A ECUATIILOR BOOLEENE SI PSEUDO-BOOLEENE

Rezumat

In lucrare autorii prezintă rezolvarea ecuaţiilor Booleene şi pseudo-booleene cu ajutorul unor matrici ternare şi cu ajutorul unor funcţii auxiliare.

Seminarul de Matematică şi Fizică
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THE LAPLACE-DIRICHLET METHOD FOR DETERMINING THE SUM OF POWER SERIES

Neamţu N.N.

In [1] and [2] are given two demonstrations for calculing the sum of the serie $\sum_{n=0}^{\infty} a(n)z^n$, $|z| < 1$, when $a(n)$ is a polynomial. In this note we give another method for determining the sum of the power serie, using the Laplace-Dirichlet transform [3] and to determin the sums of other series. We are situated in the particular case $g(n) = n$, so the Laplace-Dirichlet transform becomes

$$(1)\quad F(\lambda) = \sum_{n=0}^{\infty} a(n)e^{-n\lambda} = (\mathcal{L}(a(n))(\lambda),\ |a(n)| < Me^{s_o n},\ n \in \mathbb{N}$$

$F(\lambda)$ is a holomorphe function in the half-plain $\mathrm{Re}\,\lambda > s_o$.

We shall use the following properties of : the linearity, the delay theorem, the tehorem of differentation and integration of $F(\lambda)$ [3].

In the power serie $f(z) = \sum_{n=0}^{\infty} a(n)z^n$, $|z| < 1$ we make the substitution $z = e^{-\lambda}$, $|z| = e^{-\mathrm{Re}\lambda}$ and we obtain even the Laplace-Dirichlet transform (1) for the sequence with the term $a(n) = \sum_{k=0}^{\infty} \alpha_k n^k$, $n \in \mathbb{N}$.

Taking in view of:

$$(2)\quad (\mathcal{L}(h(n)))(\lambda) = \frac{e^{\lambda}}{e^{\lambda}-1},\ \mathrm{Re}\,\lambda > 0,\ h(k) = \begin{cases} 0 & \text{for } k < 0 \\ 1 & \text{for } k \geq 0,\ k \in \mathbb{Z} \end{cases}$$

and

$$(3)\quad (\mathcal{L}(n^k h(n)))(\lambda) = (-1)^k \frac{d^k}{d\lambda^k}\left(\frac{e^{\lambda}}{e^{\lambda}-1}\right)$$

we obtain

$$(4)\quad (\mathcal{L}(n^k))(\lambda) = \frac{e^{\lambda}}{(e^{\lambda}-1)^{k+1}} P_k(e^{\lambda})$$

where

$$(5)\quad \begin{matrix} P_0(e^{\lambda})=1, \\ \cdots\cdots\cdots \\ P_k(e^{\lambda})=k!\cdot \end{matrix} \begin{vmatrix} 1 & 1-e^{\lambda} & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{1}{k!} & \frac{1}{(k-1)!} & \frac{1}{(k-2)!} & \frac{1}{(k-3)!} & \cdots & 1 \end{vmatrix},\quad k \geq 2$$

We have $F(\lambda) = \sum_{k=0}^{p} \alpha_k \frac{e^{\lambda}}{(e^{\lambda}-1)^{k+1}} P_k(e^{\lambda})$ or

$$(6) \quad f(z) = \sum_{k=0}^{p} \alpha_k P_k(\tfrac{1}{z}) \frac{z^k}{(1-z)^{k+1}}$$

In the same way we can obtain the following relations:

$$(7) \quad \sum_{n=m}^{\infty} \binom{n}{m} z^n = \frac{z^m}{(1-z)^{m+1}}, \quad |z| < 1$$

$$(8) \quad \sum_{n=m}^{\infty} z^n \cos(n-m)\omega = z^m \frac{1-z\cos\omega}{1-2z\cos\omega+z^2}, \quad |z| < 1$$

For determining the sum of $\sum_{n=0}^{\infty} f(n)z^n$, $|z| < 1$ where

$$(9) \quad f(n) = \begin{cases} n & \text{for } 0 \leq n \leq m \\ 2m-n & \text{for } m < n < 2m \\ 0 & \text{for } n \leq 2m \end{cases}$$ we use

$$(10) \quad f(n) = nh(n) - 2(n-m)h(n-m) + (n-2m)h(n-2m)$$

the liniarity, the delay theorem, (2) and we find

$$(11) \quad \sum_{n=0}^{\infty} f(n)z^n = (1-2z^m + z^{2m}) \frac{z}{(1-z)^2}, \quad |z| < 1.$$

If $f(n) = f(n+m)$, $m \in \mathbb{N}^*$ fixed, is a periodic sequence, then we have:

$$(12) \quad \sum_{n=0}^{\infty} f(n)z^n = \frac{1}{1-z^m} \sum_{n=0}^{m-1} f(n)z^n, \quad |z| < 1$$

In the end, using

$$(13) \quad (\mathcal{L}(\operatorname{sh} n\omega))(\lambda) = \frac{1}{2}\left(\frac{e^{\lambda}}{e^{\lambda}-e^{\omega}} - \frac{e^{\lambda}}{e^{\lambda}-e^{-\omega}}\right)$$

and the integration theorem of the image, we find

$$(14) \quad \sum_{n=1}^{\infty} \frac{\operatorname{sh} n\omega}{n} z^n = \frac{1}{2} \ln \frac{e^{\omega}-z}{e^{\omega}-ze}$$

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METODA LAPLACE-DIRICHLET PENTRU DETERMINAREA SUMEI UNEI SERII DE PUTERI

Rezumat

In lucrare se dă o metodă de determinare a sumei unei serii de puteri bazată pe transformarea Laplace-Dirichlet.

Semirarul de Matematică şi Fizică
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FUNCTIONAL BICONVERGENCE

Dăianu D.

In [2, 3, 4, 5, 6, 7, 8] we realized a categorical model of the approximation schemes generalizing categorically the works of Aubin, [1], Stummel [11, 12], Marinescu [9]. We studied the functorial convergence, stability and consistency, [2,3], the inverse convergence and stability [4, 6, 7], the bistability [5], the connections between these notions and we proved in that frame the Lax-Aubin type theorems [3, 4, 7, 8]. The aim of this paper is to define the functorial biconvergence and to establish the connection with consistency and bistability.

Let (X, AX, PSX), (Y, AY, PSY) be two approximation schemes, the feature of limit unicity, $A:X \longrightarrow Y$ and $T:SX \longrightarrow SY$ two functors. The notations and notions unspecified here have been introduced in [2, 3, 4, 5, 6, 7, 8].

Definition 1. The approximation scheme (X,AX,PSX) is named complete if $AX(a) \neq 0$ for any $a \in ObX$ and $PSX(S_a, S_b) \neq \emptyset$ for any $S_a, S_b \in Ob\ PSX$, $S_a \subset AX(a)$, $S_b \subset AX(b)$ nonvoids, where $a,b \in ObX$ and $X(a,b) \neq \emptyset$.

Definition 2. The functor T is named biconvergent to A if (X,AX,PSX) and (Y,AY,PSY) are complete and:

(i) for any $a \in ObX$ and $S \subset SX$, $S \neq 0$:
$S \in ObPSX$, $S \subset AX(a)$ iff $T(S) \in ObPSY$, $T(S) \subset (AY \circ A)(a)$

(ii) for any morphism $f \in X(a,b)$ and any functor $F \in PSX(U,U)$ the diagram (1) is commutative iff the diagram (2) is commutative.

$$\begin{array}{ccc} U & \xrightarrow{F} & V \\ \cap\downarrow & (1) & \downarrow\cap \\ AX(a) & \xrightarrow[AX(f)]{} & AX(b) \end{array} \qquad \begin{array}{ccc} U & \xrightarrow{\quad F \quad} & V \\ TU\downarrow & (2) & \downarrow TV \\ (AY\circ A)(a) & \xrightarrow[(AY\circ A)(f)]{} & (AY\circ A)(b) \end{array}$$

Theorem 1. The functor T is biconvergent to A iff T

convergent and inverse convergent to A.

Proof. We suppose that T is biconvergent to A. Let $a,b \in Ob X$ and $f \in X(a,b)$. Because (X,AX,PSX) is complete $AX(a) \neq 0$, we denote $S = AX(a)$; from (i) it follows that $T(S) \in Ob PSY$ and $T(S) \subset (AY \circ A)(a)$, hence T converges to A on a. We mark $F = AX(f) \in PSX(AX(a), AX(b))$; the diagram (1) is commutative, hence (2) as well and it results that $(AX(f))(T(s)) = (AY \circ A)(f)(T(s))$, (Def,2.2, [3]), hence T converges to A on f. Let $S_a = AX(a)$, $S_b = AX(b)$; from (i) it follows that $T(S_a)$, $T(S_b) \in Ob\ PSY$ and $T(S_a) \subset (AY \circ A)(a)$, $T(S_b) \subset (AY \circ A)(b)$, the diagrams (1) and (2) are commutative for $F = AX(f)$, hence $TF(T(s)) = (AY \circ A)(f)(s)$, for any element s from S_a (Def.1, [8]); let S'_a, $S'_b \subset SX$ and $F' \in PSX(S'_a, S'_b)$ for which the diagram (1) from [8] is commutative; taking $U = S'_a$, $V = S'_b$, $F = F'$ in (ii) there results the commutativity of diagrams (1) and (2), hence T is inverse convergent to A on f.

Conversely, we suppose that T is convergent and inverse convergent to A. Let $a \in Ob\ Y$, $S \subset SX$, $S \neq 0$. If $S \in Ob\ PSX$, $S \subset AX(a)$, from the convergency of T to A on a it results that $T(S) \in OB\ PSY$, $T(S) \subset (AY \circ A)(a)$; if $T(S) \in Ob\ PSY$ and $T(S) \subset (AY \circ A)(a)$, from the inverse convergence of T to A it follows that $S \in Ob\ PSX$ and $S \subset AX(a)$, hence (i) is true. The same reason is used for proving the equivalence of commutativity of diagrams (1) and (2), hence (ii); therefore T is biconvergent to A.

Taking into account [4, 5, 6, 7, 8] and Theorem 1 we get the following characterization theorem:

Theorem 2. Let A be an isomorphism of categories. The following sentences are equivalent:

(1) T is biconvergent to A

(2) T is convergent and inverse convergent to A

(3) T is consistent with A, stable inverse stable

(4) T is consistent with A and bistable.

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BICONVERGENTA FUNCTORIALA

Rezumat

In lucrare se introduce conceptul de biconvergenţă functorială şi se stabileşte legătura cu convergenţa, convergenţa functorială inversă, cu consistenţa, stabilitatea, stabilitatea inversă şi bistabilitatea.

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LES F.V - MORPHISMES ET LES SECTIONS DANS LES FIBRES VECTORIELS (II)

Radu Fl.

Si $\eta = (E, \pi_1, M)$ et $\eta' = (E', \pi_2, M)$ sont les fibrés vectoriels on notera par $L(\eta, \eta') = (L(E,E'), \varsigma, M)$ le fibré vectoriel des applications linéaires continues de E dans E'.

Proposition 1. On considère les fibrés vectoriels (E, π_1, M) et (E', π_2, M). La section $f:M \to L(E,E')$ de classe C^k dans le fibré vectoriel $(L(E,E'), \varsigma, M)$ détermine un morphisme de fibrés vectoriels $f_1: E \to E'$ défini par: $f_1(z) = f_x(z)$ si $\pi_1(z) = x$.

Proposition 2. Soit $f:E \to E'$ un morphisme de classe C^k d'une fibré vectoriel (E, π_1, M) dans au autre fibré vectoriel (E', π_2, M). L'application $f:M \to L(E,E')$ définie de la façon suivante: $f(x) = g_x$ est une section du fibré vectoriel $(L(E,E'), \varsigma, M)$ où $g_x: E_x \to E'_x$ est la restriction de l'application $f:E \to E'$ relatif à l'espace linéaire E_x.

Démonstration. Soit (U,h) une carte locale de la variété différentiable M en $x \in M$. L'application $g:E \to E'$ étant un F.V - morphisme de classe C^k il existe les cartes vectorielles $H: \pi_1^{-1}(U) \to U \times F$ et $H': \pi_2^{-1}(U) \to U \times F'$ du fibré vectoriel (E, π_1, M) et respectiv du fibré vectoriel (E', π_2, M) telles que l'application $G:U \to L(F,F')$, $G(x) = H'_x \circ g_x \circ H_x^{-1}$ est de classe C^k donc l'application $\tilde{G} = G \circ h^{-1}$; $\tilde{G}(p) = H'_{h^{-1}(p)} \circ g_{h^{-1}(p)} \circ H^{-1}_{h^{-1}(p)}$ est de classe C^k où $p = h(x)$. Soit $\alpha: \varsigma^{-1}(U) \to U \times L(F,F')$ une carte du fibré $L(E,E')$. On considère l'application $f:M \to L(E,E')$ définie par: $f(x) = g_x$, et (U,h) et $(\varsigma^{-1}(U), (h \times I_d) \circ \alpha)$ les cartes locales de la variété M et respectiv de $L(E,E')$. En tenant compte de la définition des applications différentiables définies sur variétés différentiables on déduit que l'application $F = (h \times I_d) \circ \alpha \circ f \circ h^{-1}$

définie par $F(p) = (p,\tilde{G}(p))$ est de classe C^k. Il resulte alors que $f:M \longrightarrow L(E,E')$ est une application de classe C^k.

Proposition 3. Soient les fibrés vectoriels (E,π_1,M) et $\eta' = (E',\pi_2,M')$ l'application de classe C^k. $f_0:M \to M'$ et $f_0^*(\eta') = (f_0^*(E'),f_0^*(\pi_2),M)$ l'image réciproque du fibré vectoriel $(E',\ _2,M)$ par l'application f_0.
La section de classe C^k $g:M \to L(E,f_0^*(E'))$ dans le fibré vectoriel $(L(E,f_0^*(E),\varsigma_*,M)$ induite un F.V -morphisme de classe C^k, (f_0,f), $f:E \longrightarrow E'$ étant l'application définie par:
$f(z) = g_x(z) \in E'_{f_0(x)}$ où $\pi_1(z) = x$.

Démonstration. Pour tout $x \in M$ on considère l'application linéaire $g_x:E_x \longrightarrow E'_{f_0(x)}$. Si $H: \pi_1^{-1}(U) \to U \times F$ et $H': \pi_2^{-1}(U) \to U \times F'$, $U = f_0^{-1}(U')$ sont les cartes vectoriels du fibré vectoriel (E,π_1,M) et respectiv du fibré (E',π_2,M) alors $H_*: \pi_*^{-1}(U) \to U \times F'$, $H_{*x} = H'_{f_0(x)}$ est une carte vectorielle du fibré vectoriel $f_0^*(\eta')$ et $\alpha_1: \varsigma_*^{-1}(U) \to U \times L(F,F')$, $\alpha_1(\omega) = (x,H_{*x} \circ \omega \circ H_x^{-1})$, $\varsigma_*(\omega) = x$ est un carte vectorielle du fibré $(L(E,f_0^*(E'),\varsigma_*,M)$.

L'application $g:M \longrightarrow L(E,f_0(E'))$ étant une application de classe C^k il existe les cartes locales en $x \in M$ et respectivement en $g(x) \in L(E,f_0^*(E'))$ (U,h) et $(\varsigma_*^{-1}(U),\emptyset)$, $\emptyset = (h \circ l_d) \circ \alpha$, de la variété M et respectif $L(E,f_0(E'))$ telles que $\emptyset \circ g \circ h^{-1}$ où l'application $\emptyset \circ g:U \to L(F,F')$ est de classe C^k. Puisque $(\emptyset \circ g)(x) = (x,H_{*x} \circ g_x \circ H_x^{-1})$ il résulte que l'application $g_U:U \to L(F,F')$, $g_U(x) = (H'_{f_0(x)} \circ f_x \circ H_x^{-1})$ où $f_x = g_x$, $x \in M$ est de classe C^k. On obtient que (f_0,f) est un morphisme de fibrés vectoriels de classe C^k.

Proposition 4. Soit (f_0,f) un morphisme de classe C^k d'une fibré vectoriel (E,π_1,M) dans au autre fibré vectoriel $(E',\pi_2,M'$ L'application $F:M \to L(E,f_0^*(E'))$ définie de la faços suivante: $F(x) = f_x$, $x \in M$. ($f_x: E_x \to E'_{f_0(x)}$ étant la restriction de l'application $f:E \to E'$ relatif à $E_x = \pi_1^{-1}(x)$) est une section de classe C^k dans le fibré vectoriel $(L(E,f_0^*(E'),\varsigma_*,M)$.

Remarque. Soient $\mathcal{E}$, le $\mathcal{F}(M)$-module des sections de classe C^k dans le fibré vectoriel (E,π_1,M) et $\mathcal{E}'$ le $\mathcal{F}(M)$-module des sections de classe C^k dans le fibré (E',π_2,M). On sait que $f: \mathcal{E} \longrightarrow \mathcal{E}'$ est une application pointuelle si pour $x \in M$ et quels que soint les sections $s_1,s_2 \in \mathcal{E}$ telle

$s_1(x) = s_2(x)$ il résulte $f(s_1)(x) = f(s_2)(x)$.

Proposition 5. La section $f:M \to L(E,E')$ de classe C^k dans le fibré vectoriel $(L(E,E'), \xi, M)$ implique l'application $\mathcal{F}(M)$ - linéaire $\tilde{f} : \mathcal{E} \to \mathcal{E}'$ déterminée de façon suivante: si $s \in \mathcal{E}$ alors $f(s) \in \mathcal{E}'$ est l'application définie par: $f(s)(x) = f_x(s_x)$, $\forall x \in M$.
Si $s \in \mathcal{E}$ est de classe C^k alors $f(s) \in \mathcal{E}'$ est de classe C^k.

Démonstration. 1° $\tilde{f}(s_1 + s_2)(x) = f_x(s_1(x) + f_x(s_2(x)) = \tilde{f}(s_1)(x) + \tilde{f}(s_2)(x) = (\tilde{f}(s_1) + \tilde{f}(s_2))(x)$, $\forall x \in M$. Donc $\tilde{f}(s_1+s_2) = \tilde{f}(s_1) + \tilde{f}(s_2)$.

2° $\tilde{f}(hs)(x) = f_x(h(x)s(x)) = h(x)f_x(s(x)) = (h\tilde{f}(s))(x)$, $\forall x \in M$ alors $\tilde{f}(hs) = h\tilde{f}(s)$, $\forall h \in \mathcal{F}(M)$, $s \in \mathcal{E}$.

Soient $H: \pi_1^{-1}(U) \to U \times F$ et $H': \pi_2^{-1}(U) \to U \times F'$ les cartes vectoriels, (U,h) une carte locale de la variété M en $x \in M$ alors l'application $G: \xi^{-1}(U) \to h(U) \times L(F,F')$, $G(t) = (h(x), H'_x \circ t \circ H_x)$ si $\xi(t) = x$ est une carte locale de la variété $L(E,E')$. Puisque $f:M \to L(E,E')$ est une application de classe C^k il résulte que $G \circ f: M \to h(U) \times L(F,F')$, $(G \circ f)(x) = (h(x), H'_x \circ f_x \circ H_x^{-1})$ est de classe C^k. On obtient que $h_1: U \to L(F,F')$, $h_1(x) = H'_x \circ f_x \circ H_x^{-1}$ est de classe C^k. Si $s \in \mathcal{E}$ est de classe C^k et $H_1: \pi_1^{-1}(U) \to h(U) \times F$ est une carte locale de la variété E alors $H_1 \circ s$, $(H_1 \circ s)(x) = (h(x), H_x(s_x))$, $x \in U$ est de classe C^k donc et $h_2: U \to F$, $h_2(x) = H_x(s_x)$.

Soit $\tilde{f}(s) \in \mathcal{E}'$, $H'_1: \pi_2^{-1}(U) \to h(U) \times F'$ une carte locale de la variété E' et l'application $H'_1 \circ \tilde{f}(s)$. Lorsque $H'_1\tilde{f}(s)(x) = (h(x), h_1(x)(h_2(x)))$ en ténant compte que $\beta: L(F,F') \times F \to F'$ $\beta(T,u) = T(u)$ est une application bilinéaire continue on obtient que $f(s): U \to E'$ est de classe C^k.

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F.V - MORFISME SI SECTIUNI IN FIBRATE VECTORIALE

Rezumat

In lucrare se arată că unui F.V.-morfism al fibratului vectorial (E,π_1,M) în fibratul vectorial (E',π_2,M) (E',π_2,M') se poate asocia o secţiune în fibratul vectorial $(L(E,E'), \xi, M)$ $(L(E,f_0^*(E')), \xi_x, M)$ şi reciproc. In final se arată că o secţiune în fibratul $(L(E,E'), \xi, M))$ implică o aplicaţie $\mathcal{F}(M)$-lineară $f: \mathcal{E} \to \mathcal{E}'$

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SEMI-EINSTEIN SPACES

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In this paper semi-Einstein (S-Einstein) spaces are defined and a characteristic property, which separates these spaces from Einsten spaces (Theorem 6) is given. Furthermoore, the special case of an S-Einstein space, which is simultanously also an Einstein space, is studied.

Let V_n be a C^∞ differentiable n-dimensional manifold and (V,g) a general metrical structure on V_n, where g is a symmetric and non-degenerated tensor of the type $(0,2)$. In a local chart $\{U;x^k,\ k = 1,2,\dots,n\}$ we denote by g_{ij} the components of g, by $\left\{ {i \atop jk} \right\}$ the coefficients of the corresponding Levi-Civita connection ∇ and by Γ^i_{jk} the coefficients of an arbitrary linear connection D. We denote by R_{ij} the Ricci tensor of D, and by $R = g^{ij}R_{ij}$ the scalar curvature.

Definition 1. Let (V,g) be a metrical structure and D a fixed linear connection. If $E_{ij} = 0$, where $E_{ij} = R_{ij} - \frac{R}{n} g_{ij}$, then V_n is called a semi-Einstein (or S-Einstein) space.

We begin the study of these spaces with special cases of S-Einstein structures and expand same results of K.Yano [4].

Let D be a metrical connection ($g_{ij|k} = 0$), which is semi-symmetric:

$$(1)\quad T^i_{jk} = \omega_j \delta^i_k - \omega_k \delta^i_j\ , \quad \omega \in \Lambda^0_1(V),\ T^i_{jk} = \Gamma^i_{jk} - \Gamma^i_{kj}$$

This connection has the coefficients:

$$(2)\quad \Gamma^i_{jk} = \left\{ {i \atop jk} \right\} + \omega_j \delta^i_k - g_{jk}\omega^i$$

We can writte the relation between the curvature tensors of D and ∇:

$$(3)\quad R^h_{ijk} = r^h_{ijk} + (\omega_{i|j} - \tfrac{1}{2} g_{ij}\omega^s \omega_s)\delta^h_k - (\omega_{i|k} - \tfrac{1}{2} g_{ik}\omega^s\omega_s)\delta^h_j + g_{ij}g^{hs}(\omega_{s|k} - \tfrac{1}{2} g_{sk}\omega^r\omega_r) -$$

$$\cdot\ -g_{ik}g^{hs}(\omega_{slj} - \tfrac{1}{2}g_{sj}\omega^r\omega_r).$$

Definition 2:[1]. If we have for D e relation:

$$\omega_{ilj} - \tfrac{1}{2}g_{ij}\omega^r\omega_r = fg_{ij} \tag{4}$$

where f is a differentiable function, then D is called a semi-concircular (S-concircular) connection.

We denote by:

$$\begin{cases} T^h_{ijk} = R^h_{ijk} - \frac{R}{n(n-1)}(g_{ij}\delta^h_k - g_{ik}\delta^h_j); \\ t^h_{ijk} = r^h_{ijk} - \frac{r}{n(n-1)}(g_{ij}\delta^h_k - g_{ik}\delta^h_j) \end{cases} \tag{5}$$

the D-concircular curvature tensor and the concircular curvature tensor respectively [1]. It follows:

Theorem 1. A necessary and sufficient condition that D be S-concircular is $T^h_{ijk} = t^h_{ijk}$.

An existence theorem of S-concircular connection on V_r, which generalize Theorem 2 [4] of K.Yano is given in [3]. It follows easily:

Lemma 1. If $T^h_{ijk} = 0$, then V_n is an S-Einstein space.

Theorem 2. If there exists an S-concircular connection D, then V_n is an S-Einstein space if and only if V_n is an Einstein space. Using the Theorem 1 and the relation (3) the proof is immediately. Also it follows the:

Theorem 3. A necessary and sufficient condition that D be S-concircular is $E_{ij} = e_{ij}$, where $e_{ij} = r_{ij} - \frac{r}{n}g_{ij}$ is the Einstein tensor.

The proof is immediately.

Let consider the following assertions:

P_1: D given by (2) is S-concircular;

P_2: V_n is an S-Einstein space relative to D (D is an S-Einstein connection);

P_3: V_n is an Einstein space ($e_{ij} = 0$).

Then we have:

Theorem 4. From P_i and P_j it follows P_k ($i \neq j \neq k$).

For an Einstein space ($n \geqslant 3$) we have r = constant. For an S-Einstein space R is generally not constant. This is a very important property.

The integrability condition for (4) is:

$$R^s_{ijk}\omega_s = f_k g_{ij} - f_j g_{ik}; \quad f_k = \partial_k f \tag{6}$$

It follows that:

$$\omega^k R_{kj} = (n-1)f_j \tag{7}$$

Since D is S-concircular we have: $f = \frac{R-r}{2n(n-1)}$ and from (7) and $E_{ij} = 0$ it follows:

(8) $\quad R_j - 2R\omega_j = 0 \; ; \qquad P_j = \partial_j R$

We have:

Theorem 5. If D is S-concircular and S Einstein, then R and ω satisfies the relation (8).

From (4) it follows that $\omega_{i|j} - \omega_{j|i} = 0$ and D is a connection with the property $\omega_{i|j} = \omega_{j|i}$. The integrability condition of this equation is verified: $T^s_{ij}\omega_s = 0$, since D is semi-symmetrical. Locally it follows that there exists a differentiable function h such that $\omega_i = \frac{1}{2}\partial_i h$. From the above and from (8) it follows

(9) $\quad R = ce^h \; ; \; c = \text{const.} \neq 0 \quad (\text{if } R^h_{ijk} \neq 0).$

It follows moreover that for an S-Einstein space R is not constant . We have:

Theorem 6. For an S-Einstein space ($D \neq \nabla$), the scalar curvature R is generally not constant. Locally if V_n is an S-Einstein space relative to an S-concircular connection then, there exists a differentiable function h ($h \neq 0$), such that R is of the form (9), a function with a constante signature, the signature of C.

This theorem is very important in the physical theories, where the covector ω_i has the property $\omega_{i|j} = \omega_{j|i}$.

The question reised is the existence of such connections.

If $T^s_{ijk} = 0$, then V_n is a space with constant curvature and reciprocally: if V_n is with constant curvature then there exists a semi-concircular connection D with $T^s_{ijk} = 0$ (A generalization of a Theorem of K.Yano [4]).

But in this case, the space is S-Einstein and R satisfies (8) (locally we have (9)). The property is more generalized in [1], [3].

Corollary. If V_3 is an Einstein space, then there exists an S-Einstein S-concircular connection.

For $n > 3$ the existence problem of an S-Einstein S-concircular connection has two parts. We have:

Theorem 7. If D is an S-Einstein S-concircular connection, then V_n is an Einstein space, in which locally we have: $\omega_i = \frac{1}{2}\partial_i h$ and the equation (4) is satisfied by $f = (R-r)/2n(n-1)$: R is given by (9) and r is constant.

Theorem 8. If V_n is an Einstein space with $n \geqslant 3$, then V_n is an S-Einstein space.

For n = 3 the proof is immediately, using the above corollary.

For n > 3 it is sufficient to considere the Brinkman type [2] system of the form:

$$\begin{cases} \dfrac{\partial^2 h}{\partial x^j \partial x^i} - \left\{ {s \atop ij} \right\} \dfrac{\partial h}{\partial x^s} - \dfrac{1}{2} \dfrac{\partial h}{\partial x^i} \dfrac{\partial h}{\partial x^j} + \dfrac{1}{4} g_{ij} g^{sk} \dfrac{\partial h}{\partial x^s} \dfrac{\partial h}{\partial x^k} - \\ \qquad - \dfrac{1}{n(n-1)} (ce^h - r) g_{ij} = 0 \\ t^s{}_{ijk} \dfrac{\partial h}{\partial x^s} = 0 \; ; \quad r = \text{const.} \end{cases}$$

where h is an unknown differentiable function. Studying the complet integrability conditions it follows theorem (8), where $\omega_i = \frac{1}{2} \partial_i h$ and D is builded with this ω_i.

The general study will apear elsewhere.

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SPATII SEMI-EINSTEIN

Rezumat

In lucrare se definesc spaţiile S-Einstein şi se pune în evidenţă o proprietate care deosebeşte esenţial aceste spaţii de spaţiile Einstein (Teorema 6).
Se studiază apoi cazul special al unui spaţiu S-Einstein, care este simultan şi spaţiu Einstein.

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SEPARATION PROPERTIES IN BITOPOLOGICAL SPACES

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The notion of bitopological space is due to J.C.KELLY [1]. For the notion of weak pairwaise T_i space, i=o,1,2,3 we adopt the definitions given in [2].

The aim of this note is to characterize the weak pairwaise T_i bitopological spaces (WPT_i spaces),for i=o,1,2,3.

THEOREM 0 The bitopological space $(X; \mathcal{P}, \mathcal{Q})$ is WPT_o iff for any pair of distinct points $x,y \in X$ at least one of the following situation is true : $x \notin cl_{\mathcal{P}}\{y\} \cap cl_{\mathcal{Q}}\{y\}$ or $y \notin cl_{\mathcal{P}}\{x\} \cap cl_{\mathcal{Q}}\{x\}$.

Proof : Routine.

THEOREM 1 For an arbitrary bitopological space $(X; \mathcal{P}, \mathcal{Q})$ the following assertions are equivalent :

(1) $(X; \mathcal{P}, \mathcal{Q})$ is W P T_1;

(2) for any point $x \in X$ the set $\{x\}$ is $\mathcal{P}$-closed or $\mathcal{Q}$-closed;

(3) any set $E \subset X$ is the intersection of all $\mathcal{P}$-open or $\mathcal{Q}$-open sets including it;

(4) any point $x \in X$ is the intersection of all $\mathcal{P}$-open or $\mathcal{Q}$-open sets including it;

(5) any set $E \subset X$ is a union of $\mathcal{P}$-closed or $\mathcal{Q}$-closed sets;

(6) any nonempty subset of X includes a $\mathcal{P}$-closed or $\mathcal{Q}$-closed set.

Proof : Routine.

DEFINITION 1 If $\mathcal{F}$ is a filter on the bitopological space $(X; \mathcal{P}, \mathcal{Q})$ and $\mathcal{V}_{\mathcal{P}}(x)\left(\mathcal{V}_{\mathcal{Q}}(x)\right)$ are the family of all $\mathcal{P}$-neighborhoods (respectively $\mathcal{Q}$-neighborhoods) of the point $x \in$ and if $\mathcal{V}_{\mathcal{P}}(x) \subseteq \mathcal{F}$ (respectively $\mathcal{V}_{\mathcal{Q}}(x) \subseteq \mathcal{F}$) then we

the filter is $\mathcal{P}$-convergent (respectively) $\mathcal{Q}$-convergent to the point $x \in X$, an in this case we write $\mathcal{F} \xrightarrow{\mathcal{P}} x$ (respectively $\mathcal{F} \xrightarrow{\mathcal{Q}} x$).

DEFINITION 2 A point $x \in X$ is a $\mathcal{P}$-adherent (respectively $\mathcal{Q}$-adherent) point to the filter $\mathcal{F}$ of the bitopological space $(X; \mathcal{P}, \mathcal{Q})$ iff $x \in \bigcap_{F \in \mathcal{F}} cl_{\mathcal{P}} F$ (respectively $x \in \bigcap_{F \in \mathcal{F}} cl_{\mathcal{Q}} F$).

It is clear that if $\mathcal{F} \xrightarrow{\mathcal{P}} x$ (respectively $\mathcal{F} \xrightarrow{\mathcal{Q}} x$), then x is a $\mathcal{P}$-adherent (respectively $\mathcal{Q}$-adherent) point to $\mathcal{F}$.

THEOREM 2 For an arbitrary bitopological space $(X; \mathcal{P}, \mathcal{Q})$ the following assertions are equivalent:

(1) $(X; \mathcal{P}, \mathcal{Q})$ is WPT_2;

(2) for any $x \in X$, the set $\{x\}$ is the intersection of all $\mathcal{Q}$-closed $\mathcal{P}$-neighborhoods of X or the intersection of all $\mathcal{P}$-closed $\mathcal{Q}$-neighborhoods of x;

(3) the set $\Delta(X) = \{(x,x), x \in X\}$ is $\mathcal{PQ}$-closed or $\mathcal{QP}$-closed in the bitopological product space $(X \times X, \mathcal{PQ}, \mathcal{QP})$;

(4) any filter has at most a $\mathcal{P}$-limit or a $\mathcal{Q}$-limit;

(5) if $\mathcal{F} \xrightarrow{\mathcal{P}} x$, then x is the unique $\mathcal{P}$-adherent point to $\mathcal{F}$ and if $\mathcal{F}$ has a $\mathcal{Q}$-adherent point to $\mathcal{F}$, then this point is x. If $\mathcal{G} \xrightarrow{\mathcal{Q}} x$, then x is the unique $\mathcal{Q}$-adherent point to $\mathcal{G}$ and if $\mathcal{G}$ has a $\mathcal{P}$-adherent point to $\mathcal{G}$, then this point is x.

Proof. We show that (1) implies (2). Let $x \in X$ arbitrary. If $y \in X$ and $y \neq x$, then there exist $U \in \mathcal{P}$ and $V \in \mathcal{Q}$, $U \cap V = \emptyset$ with $x \in U$, $y \in V$ or $x \in V$ and $y \in U$.

Hence $y \notin cl_{\mathcal{Q}} U$ or $y \notin cl_{\mathcal{P}} V$. But $cl_{\mathcal{Q}} U \supset U \ni x$, so $cl_{\mathcal{Q}} U$ is a $\mathcal{Q}$-closed $\mathcal{P}$-neighborhood of x or $cl_{\mathcal{P}} V \supset V \ni x$, so $cl_{\mathcal{Q}} V$ is a $\mathcal{P}$-closed $\mathcal{Q}$-neighborhood of x and these neighborhoods do not contain y.

Therefore $y \notin \bigcap \{U \in \mathcal{V}_{\mathcal{P}}(x) \cap \overline{\mathcal{Q}}\}$ or $y \notin \bigcap \{V \in \mathcal{V}_{\mathcal{Q}}(x) \cap \overline{\mathcal{P}}\}$

We show that (2) implies (5). Consider $\mathcal{F} \xrightarrow{\mathcal{P}} x$.

Hence $\mathcal{V}_{\mathcal{P}}(x) \subseteq \mathcal{F}$. Let $y \in X$, $y \neq x$. If $\{x\} = \bigcap \{U \in \mathcal{V}_{\mathcal{P}}(x) \cap \overline{\mathcal{Q}}\}$ then there exists $U \in \mathcal{V}_{\mathcal{P}}(x)$ such that $y \notin U$. But there exists $E_U \in \mathcal{F}$, $E_U \subset U$, hence $E_U \cap (X - U) = \emptyset$, where $y \in X - U \in \mathcal{Q}$, and so $X - U \in \mathcal{V}_{\mathcal{Q}}(y)$. It results that $y \notin cl_{\mathcal{Q}} E_U$ i.e. y is not a $\mathcal{P}$-adherent point to $\mathcal{F}$. If $\{x\} = \bigcap \{U \in \mathcal{V}_{\mathcal{Q}}(x) \cap \overline{\mathcal{P}}\}$ then there exists $V \in \mathcal{V}_{\mathcal{Q}}(x)$ such that $V \not\ni y$. But there exists $E_V \in \mathcal{F}$, $E_V \subset V$, hence $E_V \cap (X - V) = \emptyset$, where $y \in X - V \in \mathcal{P}$, and so $X - V \in \mathcal{V}_{\mathcal{P}}(y)$. It results that $y \notin cl_{\mathcal{P}} E_V$, i.e. y is not a $\mathcal{Q}$-adherent point to $\mathcal{F}$.

Let now show that (5) implies (4). Suppose that there exists a filter $\mathcal{F}$ such that $\mathcal{F} \xrightarrow{\mathcal{P}} x$ and $\mathcal{F} \xrightarrow{\mathcal{Q}} y$, $x \neq y$. Then x is a

$\mathcal{P}$ - adherent an y is a $\mathcal{Q}$ -adherent point to $\mathcal{F}$ and this contradicts (5). If $\mathcal{F} \xrightarrow{\mathcal{P}} x$ and $\mathcal{F} \xrightarrow{\mathcal{P}} y$, $x \neq y$ then x and y are $\mathcal{P}$ -adherent points to $\mathcal{F}$, what is contradictory.

Now suppose (4) and admit that there exist two distinct points $x,y \in X$ with the property that any $\mathcal{P}$ -neighborhood of x meets any $\mathcal{Q}$ -neighborhood of y and that any $\mathcal{Q}$ -neighborhood x meets any $\mathcal{P}$ -neighborhood of y.

Then $\{U \cap V / U \in \mathcal{V}_{\mathcal{P}}(x), V \in \mathcal{V}_{\mathcal{Q}}(y)\}$ and $\{U \cap V / U \in \mathcal{V}_{\mathcal{Q}}(x), V \in \mathcal{V}_{\mathcal{P}}(y)\}$ are two bases generating the filters $\mathcal{F}$,respectively $\mathcal{F}'$. Hence we have $\mathcal{F} \xrightarrow{\mathcal{P}} x$, $\mathcal{F} \xrightarrow{\mathcal{Q}} y$ and $\mathcal{F}' \xrightarrow{\mathcal{Q}} x$, $\mathcal{F}' \xrightarrow{\mathcal{P}} y$, what is contradictory. It follows that (1) holds.

(3) implies (1) : Let $x,y \in X$, $x \neq y$ hence $(x,y) \notin \Delta(X)$. The set $\Delta(X)$ is $\mathcal{PQ}$ -closed ($\mathcal{QP}$ -closed).

Hence there exist $U \in \mathcal{V}_{\mathcal{P}}(x)$ and $V \in \mathcal{V}_{\mathcal{Q}}(y)$ such that $(U \times V) \cap \Delta(X) = \emptyset$. Then $U \cap V = \emptyset$.

Finally,we show that (1) implies (3).

If $(x,y) \in X \times X$ and $(x,y) \notin \Delta(X)$, then $x \neq y$ and there exists $U \in \mathcal{V}_{\mathcal{P}}(x)$, $V \in \mathcal{V}_{\mathcal{Q}}(y)$ with $U \cap V = \emptyset$.

But $U \times V$ is a $\mathcal{PQ}$ -neighborhood for (x,y) and $(U \times V) \cap \Delta(X) = \emptyset$ It results that $(x,y) \notin cl_{\mathcal{PQ}} \Delta(X)$ i.e. $\Delta(X) = cl_{\mathcal{PQ}} \Delta(X)$.

THEOREM 3 For an arbitrary bitopological space $(X; \mathcal{P}, \mathcal{Q})$ the following assertions are equivalent:

(1) $(X, \mathcal{P}, \mathcal{Q})$ is WPT_3 ;

(2) for every $\mathcal{P}$-open ($\mathcal{Q}$ -open) set D and every $x \in D$ there exists a $\mathcal{P}$-open ($\mathcal{Q}$ -open) set U_x such that $cl_{\mathcal{Q}} U_x \subset D$ ($cl_{\mathcal{P}} U_x \subset D$) ;

(3) every $\mathcal{P}$ -closed set E is the intersection of all its $\mathcal{P}$ -closed $\mathcal{Q}$ -neighborhoods and every $\mathcal{Q}$ -closed set F is the intersection of all its $\mathcal{Q}$ -closed $\mathcal{P}$ -neighborhoods;

(4) for every set $A \subset X$ and every $\mathcal{P}$ -open ($\mathcal{Q}$ -open) set D such that $A \cap D \neq \emptyset$, there exists a $\mathcal{P}$ -open ($\mathcal{Q}$ -open) set D_A such that $A \cap D_A \neq \emptyset$ and $cl_{\mathcal{Q}} D_A \subset D$ ($cl_{\mathcal{P}} D_A \subset D$).

(5) for every nonempty set $A \subset X$ and every $\mathcal{P}$ -closed ($\mathcal{Q}$-closed) set I such that $A \cap I = \emptyset$, there exist a $\mathcal{P}$ -open ($\mathcal{Q}$ -open) set D_A and a $\mathcal{Q}$-open ($\mathcal{P}$ -open) set D_I such that $A \cap D_A \neq \emptyset$ and $I \subset D_I$.

Proof (1) implies (2) ; Let $D \in \mathcal{P}$, $x \in D$. Then $X-D \in \mathcal{Q}$ $x \notin X-D$ hence there exist $U_x \in \mathcal{P}$ and $U_{X-D} \in \mathcal{Q}$ such that $x \in U_x$ $X-D \subset U_{X-D}$ and $U_x \cap U_{X-D} = \emptyset$. It results that $U_x \subset X-U_{X-D}$

(2) implies (3): Let $E \in \overline{\mathcal{P}}$ and $x \notin E$. Then $X-E \in \mathcal{P}$ and $x \in X-E$. Hence there exists $U_X \in \mathcal{P}$ such that $x \in U_X \subset cl_Q U_X \subset X-E$. The set $X-U_X$ is $\mathcal{P}$ -closed, $X-U_X \supset X- cl_{\mathcal{P}} U_X \supset E$ and $X - cl_Q U_x \in Q$. Hence $X - U_x$ is a $\mathcal{P}$ -closed Q -neighborhood of E not containing x.

It results that the intersection of all $\mathcal{P}$ -closed Q -neighborhoods of E has not points from X-E.

(3) implies (4): Let $A \subset X$ and $D \in \mathcal{P}$ such that $A \cap D \neq \emptyset$. If $x \in A \cap D$,then $x \notin X-D \in \overline{\mathcal{P}}$, hence exists a $\mathcal{P}$ -closed Q -neighborhood of X-D, V_x ,not containing x. It results that there exists $V \in Q$ so that $X-D \subset V \subset V_x$.

Let $D_A = X - V_x \in \mathcal{P}$. From $x \in D_A$ and $x \in A$ we get $A \cap D_A \neq \emptyset$ and moreover $cl_Q D_A = cl_Q (X-V_x) \subset cl_Q (X-V) = X-V \subset D$.

(4) implies (5): Let $A \subset X$, $A \neq \emptyset$ and $I \in \overline{\mathcal{P}}$, $A \cap I = \emptyset$. Then $A \cap (X-I) \neq \emptyset$ and $X-I \in \mathcal{P}$. Hence, there exist $D_A \in \mathcal{P}$ with $A \cap D_A \neq \emptyset$ and $cl_Q D_A \subset X-I$. Let $D_I = X- cl_Q D_A \supset I$.

Then $A \cap D_A \neq \emptyset$ and $I \subset D_I$.

(5) implies (1): Consider the set $A = \{x\}$ and $I \in \overline{\mathcal{P}}$, $x \notin I$. Then there exist $D_x \in \mathcal{P}$ and $D_I \in Q$ such that $\{x\} \cap D_x \neq \emptyset$ and $I \subset D_I$ hence $x \in D_x$ and $I \subset D_I$.

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PROPRIETATI DE SEPARARE IN SPATII BITOPOLOGICE

Rezumat

In lucrare se dau caracterizări ale spaţiilor T_i , i=0,1,2,3.

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ON A NON-LINEAR COMPLEX INTERPOLATION RESULT FOR n BANACH SPACES

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Our result generalizes the complex interpolation method introduced in [2] and [6]. For n = 2 this result is given in [1].

We start from some standard terminology, cf. also [2], [6] or [5], but to avoid complicated notations we content ourselves to the case n = 3, all results hold in general case.

Let us denote by

$\Omega = \{(z,w) \in \mathbb{C}^2 : \operatorname{Re} z \geq 0,\ \operatorname{Re} w \geq 0,\ 0 \leq \operatorname{Re} z + \operatorname{Re} w \leq 1\}$

$\Omega_0 = \{(z,w) \in \mathbb{C}^2 : \operatorname{Re} z > 0,\ \operatorname{Re} w > 0,\ 0 < \operatorname{Re} z + \operatorname{Re} w < 1\}$

$\mathcal{A} = \{\varphi : \Omega \to \mathbb{C} :$ continuous on bounded subsets of $\Omega\}$

Let A_0, A_1, A_2 be given Banach spaces, which are simultaneous continuously embedded in the same Hausdorff space. For $\varphi \in \mathcal{A}$ we let $\mathcal{F}(A_0, A_1, A_2, \varphi)$ denote the set of functions $f : \Omega \to A_0 + A_1 + A_2$ which satisfy the conditions:

a) f is continuous on bounded subsets of Ω

b) f is analytic on Ω_0

c) $(\forall)\ t, \tau \in \mathbb{R}$, $f(it, i\tau) \in A_0$, $f(1+it, i\tau) \in A_1$, $f(it, 1+i\tau) \in A_2$

and

$\|f(it,i\tau)\|_{A_0} \leq \text{const}\,|\varphi(it,i\tau)|$, $\|f(1+it,i\tau)\|_{A_1} \leq \text{const}\,|\varphi(1+it,i\tau)|$

$\|f(it,1+i\tau)\|_{A_2} \leq \text{const}\,|\varphi(it,1+i\tau)|$. Let T be a distribution with compact suport in Ω (i.e., $T \in \mathcal{E}'(\Omega)$). For $\varphi \in \mathcal{A}$ the complex interpolation space $(A_0, A_1, A_2)_{T,\varphi}$ is the Banach space of all $a \in A_0 + A_1 + A_2$ for which there is an $f \in \mathcal{F}(A_0, A_1, A_2, \varphi)$ satisfying $T(f) = a$. The norm of $a \in (A_0, A_1, A_2)_{T,\varphi}$ is $\|a\|_{T,\varphi} = \inf\{\|f\|_{\mathcal{F}}, T(f) = a\}$, where

$$\|f\|_{\mathcal{F}} = \max\left\{\sup_{t,\tau} \frac{\|f(it,i\tau)\|_{A_0}}{|\varphi(it,i\tau)|},\ \sup_{t,\tau} \frac{\|f(1+it,i\tau)\|_{A_1}}{|\varphi(1+it,i\tau)|},\right.$$

$$\sup_{t,\tau} \frac{\| f(it,1+i\tau)\|_{A_2}}{|\xi(it,1+i\tau)|} \Big\}$$

Theorem. Let A_0, A_1, A_2 and B_0, B_1, B_2 be given Banach spaces. Assume that (non-linear) operator L fulfills

i) $f \in \mathcal{F}(A_0, A_1, A_2, \xi)$ implies $Lf \in \mathcal{F}(B_0, B_1, B_2, \xi)$

ii) $\|La\|_{B_0} \le |\xi(it,i\tau)| \mu_0\left(\frac{\|a\|_{A_0}}{|\xi(it,i\tau)|}\right)$;

$$\|La\|_{B_1} \le |\xi(1+it,i\tau)| \mu_1\left(\frac{\|a\|_{A_1}}{|\xi(1+it,i\tau)|}\right)$$

$$\|La\|_{B_2} \le |\xi(it,1+i\tau)| \mu_2\left(\frac{\|a\|_{A_2}}{|\xi(it,1+i\tau)|}\right), \text{ where } \mu_j; \ (j=0,1,2)$$

are continuous, increasing and positive on $\mathbb{R}_+$.

Then $a \in (A_0, A_1, A_2)_{T,\xi}$ implies $La \in (B_0, B_1, B_2)_{T,\xi}$

$$\|La\|_{(B_0,B_1,B_2)_{T,\xi}} \le \max(\mu_0(\|a\|_{T,\xi}), \mu_1(\|a\|_{T,\xi}), \mu_2(\|a\|_{T,\xi}))$$

Proof. Let be $a \in (A_0, A_1, A_2)_{T,\xi}$. Then $a = T(f)$ for some $f \in \mathcal{F}(A_0, A_1, A_2, \xi)$ and $\|f\|_{\mathcal{F}(A_0,A_1,A_2,\xi)} \le \|a\|_{T,\xi} + \varepsilon$,

$\varepsilon > 0$ being arbitrary. Let g be the function defined by $g = Lf$. Then g belongs to $\mathcal{F}(B_0, B_1, B_2, \xi)$ and

$$\|La\|_{(B_0,B_1,B_2)_{T,\xi}} \le \|Lf\|_{\mathcal{F}(B_0,B_1,B_2,\xi)} =$$

$$= \max\Big\{ \sup_{t,\tau} \frac{\|Lf(it,i\tau)\|_{B_0}}{|\xi(it,i\tau)|}, \sup_{t,\tau} \frac{\|Lf(1+it,i\tau)\|_{B_1}}{|\xi(1+it,i\tau)|},$$

$$\sup_{t,\tau} \frac{\|Lf(it,1+i\tau)\|_{B_2}}{|\xi(it,1+i\tau)|} \Big\} \le \max(\mu_0(\|a\|_{T,\xi}+\varepsilon), \mu_1(\|a\|_{T,\xi}+\varepsilon),$$

$$\mu_2(\|a\|_{T,\xi}+\varepsilon))$$

When ε tends to zero we obtain

$$\|La\|_{(B_0,B_1,B_2)_{T,\xi}} \le \max_{j=0,1,2} (\mu_j(\|a\|_{T,\xi})$$

Remarks. 1) When $T = \delta(\theta_1,\theta_2)$ (Dirac's distribution) $((\theta_1,\theta_2) \in [0,1] \times [0,1])$ and $(\forall)\ (z,w) \in \Omega$, $\xi(z,w) = 1$ we obtain the complex interpolation method for n ($n = 3$) Banach spaces introduced by Favini [2]. In this case we take

$$\xi(z,w) = [\mu_0(\|a\|_{\delta,1})]^{z+w-1} [\mu_1(\|a\|_{\delta,1})]^{-z} [\mu_2(\|a\|_{\delta,1})]^{-w} L(f(z,w))$$

Then we have:

$$\|\xi(it,i\tau)\|_{B_0} \le [\mu_0(\|a\|_{\delta,1})]^{-1} \mu_0(\|a\|_{\delta,1}+\varepsilon)$$

$$\|\xi(1+it,i\tau)\|_{B_1} \le [\mu_1(\|a\|_{\delta,1})]^{-1} \mu_1(\|a\|_{\delta,1}+\varepsilon)$$

$$\|g(it,1+i\tau)\|_{B_2} \le \left[\mu_2(\|a\|_{\delta,1})\right]^{-1} \mu_2(\|a\|_{\delta,1}+\varepsilon)$$

$$g(\theta_1,\theta_2) = \left[\mu_0(\|a\|_{\delta,1})\right]^{\theta_1+\theta_2-1} \left[\mu_1(\|a\|_{\delta,1})\right]^{-\theta_1} \left[\mu_2(\|a\|_{\delta,1})\right]^{-\theta_2} La$$

Since $Lf \in \mathcal{F}(B_0,B_1,B_2,1)$ we have $La \in (B_0,B_1,B_2)_{\delta,1}$ and

$$\|La\|_{(B_0,B_1,B_2)_{\delta,1}} = \left[\mu_0(\|a\|_{\delta,1})\right]^{1-\theta_1-\theta_2} \left[\mu_1(\|a\|_{\delta,1})\right]^{\theta_1} \cdot$$

$$\left[\mu_2(\|a\|_{\delta,1})\right]^{\theta_2} \|g(\theta_1,\theta_2)\|_{\delta,1} \le \left[\mu_0(\|a\|_{\delta,1})\right]^{1-\theta_1-\theta_2} \cdot$$

$$\left[\mu_1(\|a\|_{\delta,1})\right]^{\theta_1} \left[\mu_2(\|a\|_{\delta,1})\right]^{\theta_2} \|g\|_{\mathcal{F}(B_0,B_1,B_2,1)} \le$$

$$\le \left[\mu_0(\|a\|_{\delta,1})\right]^{1-\theta_1-\theta_2} \left[\mu_1(\|a\|_{\delta,1})\right]^{\theta_1} \left[\mu_2(\|a\|_{\delta,1})\right]^{\theta_2} \cdot$$

$$\cdot \max_{j=0,1,2} \left(\left[\mu_j(\|a\|_{\delta,1})\right]^{-1} \mu_j(\|a\|_{\delta,1}+\varepsilon)\right)$$

When ε tends to zero we obtain:

$$\|La\|_{(B_0,B_1,B_2)_{\delta,1}} \le \left[\mu_0(\|a\|_{\delta,1})\right]^{1-\theta_1-\theta_2} \left[\mu_1(\|a\|_{\delta,1})\right]^{\theta_1} \left[\mu_2(\|a\|_{\delta,1})\right]^{\theta_2}$$

2) When L is linear and bounded from A_j to B_j the result is given in [6].

3) When $T = \delta(\theta)$; $\theta \in [0,1]$; $n = 2$ and $\varrho(z) = 1$ the result is given in [1].

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ASUPRA UNUI REZULTAT NELINIAR DIN INTERPOLAREA COMPLEXA PENTRU n SPATII BANACH

Rezumat

In lucrare se studiază comportarea operatorilor neliniari, supuşi unor restricţii, între spaţii Banach obţinute cu metoda complexă de interpolare introdusă în [6] şi [7].

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A CHARACTERIZATION OF GAMMA FUNCTION

Găvruţa P.

We denote by Γ the Gamma function of Euler: $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$, $x > 0$. It is well-known that $\ln\Gamma$ is a convex function on $(0,\infty)$. H.Bohr and I.Mollerup [3], [2] proved that if $f: (0,\infty) \longrightarrow (0,\infty)$ has the properties

1) $f(1) = 1$;

2) $f(x+1) = xf(x)$, $x \in (0,\infty)$;

3) $\ln f$ is a convex function on $(0,\infty)$, then $f = \Gamma$.

Other characterizations of Gamma function by means of functional equations was proved in [1], [4], [5].

Our aim is to establish the following

Theorem. The function $F(x) = \left(\Gamma(x+1)\right)^{1/x}$, $x > 0$ is a (strictly) logaritmic - concave.

If $f: (0,\infty) \longrightarrow (0,\infty)$ has the properties:

1. $f(1) = 1$;

2. $f(x+1) = xf(x)$, $x \in (0,\infty)$;

3. the function $G(x) = \left(f(x+1)\right)^{\frac{1}{x}}$, $x > 0$ is, logaritmic - concave, then $f = \Gamma$.

Proof. Let be

$$H(x) = \ln F(x) = \frac{\ln\Gamma(x+1)}{x}, \quad x > 0.$$

We will prove that H is a concave function on $(0,\infty)$. If

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}, \quad x > 0$$

is the Digamma function, we have $\psi''(x) < 0$, $x > 0$ and

$$H''(x) = \frac{x^2\psi'(x+1) - 2x\psi(x+1) + 2\ln\Gamma(x+1)}{x^3}.$$

Let be

$$L(x) = x^2\psi'(x+1) - 2x\psi(x+1) + 2\ln\Gamma(x+1),\ x>0.$$

We have

$$L'(x) = x^2\psi''(x+1) < 0,\ x > 0.$$

This implies that L is a strictly increasing function. Hence $L(x) < L(0) = 0$, $x > 0$. It follows that $H''(x) < 0$, $x > 0$. Therefore, H is a (strictly) concave function on $(0,\infty)$.

Now, let be f a function with the properties 1, 2, 3. Since the function

$$M(x) = \ln f(x) = \frac{\ln f(x+1)}{x},\quad x > 0$$

is concave, it follows

$$(1)\qquad \frac{M(x_2) - M(x_1)}{x_2 - x_1} \geqslant \frac{M(x_3) - M(x_2)}{x_3 - x_2}$$

for $x_1 < x_2 < x_3$. For $0 < x < 1$, from (1) we have

$$M(n) - M(n-1) \geqslant \frac{M(x+n) - M(n)}{x} \geqslant M(n+1) - M(n).$$

for $n \in \mathbb{N}$, $n \geqslant 2$, hence

$$(2)\qquad \left\{\frac{(f(n+2))^{\frac{1}{n+1}}}{(f(n+1))^{\frac{1}{n}}}\right\}^x \leqslant \frac{(f(x+n+1))^{\frac{1}{x+n}}}{(f(n+1))^{\frac{1}{n}}} \leqslant \left\{\frac{(f(n+1))^{\frac{1}{n}}}{(f(n))^{\frac{1}{n-1}}}\right\}^x$$

From (2) it follows

$$(3)\qquad (f(n+1))^{\frac{x+n}{n}}\left\{\frac{(f(n+2))^{\frac{1}{n+1}}}{(f(n+1))^{\frac{1}{n}}}\right\}^{x(x+n)} \leqslant f(x+n+1) \leqslant$$

$$\leqslant (f(n+1))^{\frac{x+n}{n}}\left\{\frac{(f(n+1))^{\frac{1}{n}}}{(f(n))^{\frac{1}{n-1}}}\right\}^{x(x+n)}.$$

But from 1 and 2 we have $f(n) = (n-1)!$ and

$$f(x+n+1) = f(x)x(x+1)\ldots(x+n+1)(x+n),\quad x > 0,$$

hence (3) implies

$$(4)\qquad \left(\frac{\sqrt[n+1]{(n+1)!}}{\sqrt[n]{n!}}\right)^{x^2-x} \frac{n!(n+1)^x}{x(x+1)\ldots(x+n)} \leq f(x) \leq$$

$$\leq \frac{n!(n+1)^x}{x(x+1)\ldots(x+n)} \left(\frac{\sqrt[n]{n!}}{\sqrt[n-1]{(n-1)!}}\right)^{x^2+x} \left(\frac{n}{n+1}\right)^x .$$

From (4) with the formulas

$$\Gamma(x) = \lim_{n\to\infty} \frac{n!(n+1)^x}{x(x+1)\ldots(x+n)}$$

and

$$\lim_{n\to\infty} \frac{\sqrt[n]{n!}}{\sqrt[n-1]{(n-1)!}} = 1$$

we have $f(x) = \Gamma(x)$, $x \in (0,1)$. The proof in the case $x \in (1,\infty)$ it follows from 2. See also [5].

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O CARACTERIZARE A FUNCTIEI GAMMA

Rezumat

Se prezintă o caracterizare nouă a funcţiei Gamma a lui Euler.

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ABOUT A PROBABILISTIC INTEGRAL

Lipovan O.

The purpose of this paper is to develop an integration theory for positive functions on a set S, with respect to a measure defined on a σ-algebra $\mathcal{A}$ of subsets of S with values in $\mathfrak{R}_+^+$ the space of all distribution functions.

1. Notations and basic assumptions. Let $\mathcal{A}$ be an algebra of subsets of a set S. The terminology and notation are sames in [3] We will denote by $\mathfrak{R}^+$ the family of all distribution functions, (i.e. functions $F : R \longrightarrow [0,1]$, non decreasing, left continuous and $F(0) = 0$), by $\mathfrak{R}_+^+ = \{ F \in \mathfrak{R}^+, \sup_{x>0} F(x) = 1 \}$. The mapping $\tau_M : \mathfrak{R}_+^+ \times \mathfrak{R}_+^+ \longrightarrow \mathfrak{R}_+^+$, defined by :

$$\tau_M(F,G)(x) = \sup_{u+v=x} Min\,(F(u),G(v))$$

is a triangle function, and $(\mathfrak{R}_+^+, \tau_M)$ is a commutative semigroup with unity H_o. By H_o we denote the function of $\mathfrak{R}_+^+$ for which $H(x)=1$ for every $x>0$. We denote by $(\mathfrak{R}_+^+, \tau_M, d_\chi)$ the uniform topological semigroup with respect to the modified Levy's metric (sibley) d_χ .

If j denotes the identity function on $[-\infty, \infty]$, then for any F in $\mathfrak{R}_+^+$ and any $a>0$, the distribution function in $\mathfrak{R}_+^+$ whose value for any $x>0$ is $F(x/a)$ may be conveniently denoted by $F(j/a)$. In [10] the authors showed that:

$$(F(j/a),F(j/b)) = F(j/a+b) \ , \ F \in \mathfrak{R}_+^+ \ , a,b>0$$

In this paper we will make the convention : $F(j/o) = H_o$, and $F(j/\infty) = H_o$ and we denote by

$$\tilde{\tau}_M(F_i) = \tau_M[F_1, \tau_M(F_2, \ldots, \tau_M(F_{n-1}, F_n), \ldots]$$

Let T be a t-norm, i.e. an operation on $[0,1]$ such that $([0,1], T)$ is an order semigroup with unit 1.

Definition 1.1. We say that a mapping $\gamma : \mathcal{A} \longrightarrow \mathfrak{R}^+$ is a probabilistic submeasure on $\mathcal{A}$ with respect to T if :

(i) $\gamma_\phi = H_0$; (ii) $A \subset B$ implies $\gamma_A(x) \geq \gamma_B(x)$, for every $x > 0$, $A, B \in \mathcal{A}$
(iii) for each A, B in $\mathcal{A}$, $\gamma_{A\cup B}(x+y) \geq T(\gamma_A(x), \gamma_B(y)), x > 0, y > 0$

For $\varepsilon > 0, \lambda > 0$, let $\mathcal{U}(\varepsilon,\lambda) = \{A \in \mathcal{A}, \gamma_A(\varepsilon) > 1 - \lambda\}$. If t-norm T is continuous, then the family $\mathcal{U} = \{\mathcal{U}(\varepsilon,\lambda) ; \varepsilon > 0, \lambda > 0\}$ is a normal base of neighbourhoods of ϕ for a Fréchet-Nikodym topology τ_γ, and $(\mathcal{A}, \Delta, \cap, \tau_\gamma)$ is a topological ring [2].

Definition 1.2. If $\gamma : \mathcal{A} \longrightarrow \mathcal{R}^+$ is a probabilistic submeasure with respect to a continuous t-norm, then the outer submeasure induced by γ denoted γ^* is defined by $\gamma^* : \mathcal{P}(S) \longrightarrow \mathcal{R}^+$ where

$$\gamma^*_A(x) = \sup\{\gamma_E(x) ; A \subseteq E \in \mathcal{A}\}, \quad A \subset S.$$

Let $\Gamma = \{\gamma_i\}_{i\in I}$ be a non empty family of Menger submeasures on $\mathcal{A}$ with the t-norms $T_i, i \in I$ continuous. Let there be:
$\mathcal{B}_\Gamma = \{\mathcal{U}_{k,\varepsilon,\lambda} ; k$ finite $\subset I, \varepsilon > 0, \lambda > 0\}$, where $\mathcal{U}_{k,\varepsilon,\lambda} = \{A \in \mathcal{P}(S) ; \gamma^*_A(\varepsilon) > 1-\lambda, i \in k\}$
By [6] there exists a unique topology τ_Γ on $\mathcal{P}(S)$, so that $\mathcal{P}(S)(\Gamma) = (\mathcal{P}(S), \Delta, \cap, \tau_\Gamma)$ is a topological ring, and $\mathcal{B}_\Gamma$ is a normal base of neighbourhoods of ϕ for this topology. We denote by $E_\alpha \xrightarrow{\Gamma} E$, the convergence from $\mathcal{P}(S)(\Gamma)$. We say that the set $E \in \mathcal{A}$ is Γ-finite (Γ-negligible) if for any $i \in I$, $\sup_{x>0} \gamma^i_E(x) = 1$ (resp. $\gamma^i_E(x) = H(x), x > 0$).

Definition 1.3. We say that a mapping $\mu : \mathcal{A} \longrightarrow \mathcal{R}^+_\nu$ is a probabilistic measure (in the meaning of Šerstnev) on $\mathcal{A}$ if:
(M_1) $\mu(A) = H_0, A \in \mathcal{A}$ iff $A = \phi$
(M_2) $A, B \in \mathcal{A}$, $A \cap B = \phi$ implies $\mu(A \cup B) = \tau_M[\mu(A), \mu(B)]$
We will denote by : $\mu(A) = \mu_A$ ($\mu(A)(x) = \mu_A(x), x > 0$)

Remark 1.4. Every real measure may be regarded as a probabilistic measure of a special kind. If $\nu : \mathcal{A} \longrightarrow R_+$ is a measure, then using the function H_0, a probabilistic measure on $\mathcal{A}$ can be defined by $\mu(A)(x) = H_0(x - \nu(A))$. The number $\mu(A)(x) = \mu_A(x)$ is interpreted as the probability that the measure $\nu(A)$ of A is less than x.

Let $[0,\infty]$ with the usual definition of + and usual topology. Then $([0,\infty], \mathfrak{S})$ is a compact and uniform semigroup where $\mathfrak{S}$ is the uniform structure on $[0,\infty]$.

We will fix a probabilistic measure $\mu : \mathcal{A} \longrightarrow \mathcal{R}^+_\nu$ and we will choose a family of Menger submeasures Γ_μ so that the following continuity axioms are satisfied:

A_1/ For every $A \in \mathcal{A}$ and every entourage $\mathcal{W}$ from $\mathcal{R}^+_\nu$ to exists the entourage $\sigma \in \mathfrak{S}$ with the following property: for any $n \in N$, if $\{(a_i, b_i)\}_{i=1}^n$ is from σ and if $\{E_i\}_{i=1}^n$ is a sequence of sets disjoint two by two from $\mathcal{A}$, then :

$$\left(\overset{m}{\underset{i=1}{\tau_M}} \mu_{E_i \cap A}(j/a_i) \, , \, \overset{m}{\underset{i=1}{\tau_M}} \mu_{E_i \cap A}(j/b_i) \right) \in \mathcal{W}$$

A_2/ For any $a \in [0,\infty]$, $\lim\limits_{\substack{E \xrightarrow{\Gamma_\mu} \phi \\ E \in \mathcal{S}}} \mu_E(j/a) = H_0$

In the sequel we will define the integrability of functions from $[0,\infty]^S$. To K finite $\subset I$, $\varepsilon > 0$, $\lambda > 0$ and the entourage $\sigma \in \mathfrak{S}$, we will associate the set:

$$\mathcal{W}_K(\varepsilon,\lambda;\sigma) = \left\{ (f,g) \in [0,\infty]^S \times [0,\infty]^S ; \ \gamma^{i*}_{\{s \in S ; (f(s),g(s)) \notin \sigma\}}(\varepsilon) > 1-\lambda, i \in K \right\}$$

The family $\left\{ \mathcal{W}_K(\varepsilon,\lambda;\sigma) \right\}_{\substack{\varepsilon>0,\lambda>0 \\ \sigma \in \mathfrak{S}}}$ forms a fundamental system of entourages for an uniform structure $\mathbb{U}_{\Gamma_\mu}$ on $[0,\infty]^S$.

We denote $[0,\infty]^S(\Gamma_\mu) = ([0,\infty]^S, \mathbb{U}_{\Gamma_\mu})$. If $\{f_\alpha\}$ is a generalized sequence from $[0,\infty]^S$ and if $\{f_\alpha\}$ converges to f in $[0,\infty]^S(\Gamma_\mu)$, we say that $\{f_\alpha\}$ converges to f in Γ_μ submeasures and we denote $f_\alpha \xrightarrow{\Gamma_\mu} f$

2. Integrable functions

Definition 2.1. It is said that a $\mathcal{S}$-step function, $f \in [0,\infty]^S$ is Γ_μ-integrable if:

a) f takes a finite number of distinct values $x_1, x_2, \ldots, x_n$, respectively on the sets $E_1, E_2, \ldots, E_n$

b) For $i=1,2,\ldots,n$, $x_i = 0$, it follows that E_i is Γ_μ finite (if $x_i = \infty$ it results that E_i is Γ_μ negligible)

For $E \in \mathcal{S}$, Γ_μ-integral of f on E is by definition:

$$\int_E f \, d\mu = \overset{m}{\underset{i=1}{\tau_M}} \mu_{E_i \cap E}(j/x_i)$$

We denote by $\mathcal{E}(\Gamma_\mu)$ the Γ_μ-integrable step functions set.

Theorem 2.2.

(i) Relatively to the addition operation $(f+g)(s) = f(s)+g(s)$ $\mathcal{E}(\Gamma_\mu)$ is a subsemigroup of $[0,\infty]^S$.

(ii) For $E \in \mathcal{S}$, the map $f \longrightarrow \int_E f \, d\mu$ of $\mathcal{E}(\Gamma_\mu)$ in $\mathcal{R}_v^+$ is additive:

$$\int_E (f+g) d\mu = \tau_M \left(\int_E f \, d\mu \, , \int_E g \, d\mu \right) \quad f,g \in \mathcal{E}(\Gamma_\mu)$$

(iii) For $f \in \mathcal{E}(\Gamma_\mu)$, the map $E \longrightarrow \nu(E)$, (denoted $\nu_E = \nu(E)$) $E \in \mathcal{S}$, $\nu_E = \int_E f \, d\mu$, is additive:

$$\nu_{\bigcup_{i=1}^m E_i} = \overset{m}{\underset{i=1}{\tau_M}} (\nu_{E_i}) \, , E_i \cap E_j = \phi \, , i \neq j \, ;$$

and $\nu_\phi = H_0$

(iV) For $f \in \mathcal{E}(\Gamma_\mu)$, $\lim\limits_{E \xrightarrow{\Gamma_\mu} \phi} \nu_E = \lim\limits_{E \xrightarrow{\Gamma_\mu} \phi} \int_E f \, d\mu = H_0$

Proof. It follows from definition 2.1. and hypothesis A_1/

Definition 2.3. The function $f \in [0,\infty]^S$ is called Γ_μ-integrable if there exists a generalized sequence $\{f_\alpha\}$ from $\mathcal{E}(\Gamma_\mu)$ so that $f_\alpha \xrightarrow{\Gamma_\mu} f$ and $\left\{ \int_E f_\alpha \, d\mu \right\}$ is a generalized Cauchy sequence in $\mathcal{R}_v^+$ uniformly in $E \in \mathcal{S}$. Then the Γ_μ-integral of f on $E \in \mathcal{S}$

is the element from $\widehat{\mathcal{R}}_\nu^+$ the completing of $\mathcal{R}_\nu^+$ defined by :

$$\int_E f\,d\mu = \lim_\alpha \int_E f_\alpha\,d\mu$$

We denote by $\mathcal{L}(\Gamma_\mu)$ the set of Γ_μ-integrable functions from $[0,\infty]^S$. It is obvious that $\mathcal{E}(\Gamma_\mu) \subseteq \mathcal{L}(\Gamma_\mu)$. The results of the theorem 2.2. are also valid for the functions from $\mathcal{L}(\Gamma_\mu)$.

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ASUPRA UNEI INTEGRALE PROBABILISTE

Rezumat

In lucrare se prezintă o teorie a integrării funcţilor pozitive prin raport cu o măsură probabilistă în sensul lui Serstnev,utilizînd familii de submăsuri probabiliste şi inelele topologice de mulţimi asociate acestora.

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L'ETUDE SUR LA REPARTITION DES ZEROS DES FONCTIONS ANALYTIQUES (IV)

Tudor Gh.

1. Dans quelques travaux antérieurs (voir par example [1], [2], [3], etc.), l'auteur de cette note considère certaines fonctions d'une variable complexe, pour qurlles, il fait un étude sur la distribution des zéros des cettes foctions.

2. Dans ce travail, nous continuons ces problèmes et établisons quelques limites inférieures pour les zéros de la fonction

$$(1) \qquad f(z) = c_0 + c_1 z + c_2 z^2 + \ldots + c_n z^n + \ldots$$

$$c_n \in \mathbb{C}, \quad n = 1,2,\ldots, \quad c_0 > 0, \quad |z| < R.$$

Soit $\mathcal{Z}_f$, l'ensemble des zéros de cette fonction.

Théorème 1. Soit une série aux termes positifs

$$(u): \sum_1^\infty u_n \leq u_0 < +\infty$$

Pour tout $z_0 \in \mathcal{Z}_f$, on a l'inégalité

$$(2) \qquad |z_0| \geq \Omega[f] \overset{df}{=} \inf_{n \geq 0} \frac{u_{n+1}}{u_n} \left| \frac{c_n}{c_{n+1}} \right|$$

Démonstration. En posant $z = re^{i\varphi}$, $c_n = |c_n| e^{i\varphi_n}$, on peut écrire:

$$\operatorname{Re} f(z) = \lim_{n \to \infty} K_n(r, \varphi, \bar{c}), \quad \text{où}$$

$$K_n(r, \varphi, \bar{c}) = \Delta_0(r) u_0 + \sum_{k=1}^{n-1} \Delta_k(r) \left(u_0 + \sum_{\nu=1}^{k} u_\nu \cos(\nu\varphi + \varphi_n) \right)$$

$$\Delta_k(r) = |c_k| r^k/u_k - |c_{k+1}| r^{k+1}/u_{k+1}, \quad k = 0,1,2,\ldots$$

En supposant que $\Delta_k(r) > 0$, $k = 0,1,2,\ldots$, il resulte que $\mathrm{Re}\, f(z) > 0$, ce qui prouve le théorème.

Théorème 2. Soit la série suivante:

$$(\alpha) \qquad \sum_1^\infty \alpha_n t^n \leq \alpha_0(t), \; t \in [0,\lambda), \; \alpha_n > 0, \; n = 1,2,\ldots$$

où $\alpha_0(t)$ est une fonction bornée sur $[0,\varepsilon] \subset [0,\lambda)$. Si la fonction $\varphi(t) = t/\alpha_0(t)$ n'a pas des points d'extrème dans $[0,\lambda)$ et $\lim_{t\to\lambda-} \varphi(t) = 0$, alors, tout $z_0 \in \mathcal{Z}_f$, satisfait à l'inégalité

$$(3) \qquad |z_0| \geq t_0 \inf_{n\geq 1} \frac{\alpha_{n+1}}{\alpha_n} \left|\frac{c_n}{c_{n+1}}\right|$$

où to est la racine de l'équation

$$(4) \qquad \alpha_1 \left|\frac{c_0}{c_1}\right| \varphi(t) = t \inf_{n\geq 1} \frac{\alpha_{n+1}}{\alpha_n} \left|\frac{c_n}{c_{n+1}}\right|, \quad t \in (0,\lambda)$$

Démonstration. En vertu du théorème 1, on a l'inégalité:

$$(5) \qquad |z_0| \geq \sup_{t\in(0,\lambda)} \inf_{n\geq 1} \left(\frac{\alpha_1 t}{\alpha(t)} \left|\frac{c_0}{c_1}\right|, \frac{\alpha_{n+1}}{\alpha_n} \left|\frac{c_n}{c_{n+1}}\right| t\right)$$

Il suffit observer que le point $t \in (0,\lambda)$ pour qu'on ait sup signalé ci-dessus, satisfait à l'équation (4).

Considèrons maintenant comme la série (α), la série particulière:

$$(6) \qquad \frac{\alpha}{1!} t + \frac{\alpha(\alpha+1)}{2!} t^2 + \ldots + \ldots \leq (1-t)^{-\alpha} - 1; \; \alpha > 0, \; t \in [0,1)$$

A l'aide du théoreme 2, on obtient

$$(7) \qquad |z_0| \geq \ell(\alpha) \overset{df}{=} h(\alpha)\left[1 - \left(1 + \frac{\alpha}{h(\alpha)}\left|\frac{c_0}{c_1}\right|\right)^{-\frac{1}{\alpha}}\right]$$

où

$$h(\alpha) = \inf_{n\geq 1} \frac{n+\alpha}{n+1} \left|\frac{c_n}{c_{n+1}}\right|$$

En prenant pour α quelques valeurs particulières, nous

pouvons obtenir de (7), certaines limites inférieures pour les modules des zéros de la fonction (1).

Théorème 3. Pour tout $z_0 \in \mathcal{Z}_f$, on a les inégalités:

$$(8) \qquad \frac{1}{|z_0|} \leq \frac{1}{\omega_{-1}[f]} + \frac{1}{\lambda}, \quad \omega_{-1}[f] = \inf_{n \geq 1} \left| \frac{c_n}{c_{n+1}} \right|,$$

$$\lambda = \left| \frac{c_0}{c_1} \right|$$

$$(9) \qquad |z_0| \geq \omega_0[f](1 - e^{-\lambda/\omega_0[f]}), \quad \omega_0[f] =$$

$$= \inf_{n \geq 1} \frac{n}{n+1} \left| \frac{c_n}{c_{n+1}} \right|$$

Démonstration. Il suffit faire $\alpha \to 1$ respectivement $\alpha \to 0$ dans la limite $\ell(\alpha)$.

Corollaire 1. Si la suite $(|c_n|/|c_{n+1}|)$ $n \geq 1$ est non-croissante, alors, pour tout $z_0 \in \mathcal{Z}_f$, on a l'inégalité

$$(10) \qquad \frac{1}{|z_0|} \leq \frac{1}{R} + \frac{1}{\lambda}.$$

Si la suite $(n|c_n|/(n+1)|c_{n+1}|)_{n \geq 1}$ est non-croissante, alors, tout $z_0 \in \mathcal{Z}_f$, satisfait à l'inégalité

$$(11) \qquad |z_0| \geq R(1 - e^{-\lambda/R}).$$

Théorème 4. (d'approximation). Si on a les conditions

$$(A) \qquad \left(\frac{n}{n+1} \left| \frac{c_n}{c_{n+1}} \right| \right)_{n \geq 1} \text{ - non - croissante et } \left| \frac{c_0}{c_1} \right| \geq R \ln \frac{R}{\varepsilon}$$

où

$$(B) \qquad \left(\left| \frac{c_n}{c_{n+1}} \right| \right)_{n \geq 1} \text{ - non - croissante et } \left| \frac{c_0}{c_1} \right| \geq \frac{R(R - \varepsilon)}{\varepsilon},$$

$$0 < \varepsilon < R,$$

alors, tout $z_0 \in \mathcal{Z}_f$ sont situés dans la courrone $0 < R - \varepsilon < |z| < R$.

Démonstration. On peut utiliser le théorème 1. Dans le cas (A) $\omega_0[f] = R$, et dans le cas (B), $\omega_{-1}[f] = R$. Pour ε - petit, on peut approximer $|z_0| \simeq R$ pour tout $z_0 \in \mathcal{Z}_f$, à l'érreure $< \varepsilon$.

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STUDIUL ASUPRA REPARTITIEI ZEROURILOR FUNCTIILOR ANALITICE

Rezumat

In această lucrare se demonstrează o teoremă generală, 1, prin care se stabilesc limite inferioare pentru modulele zerourilor funcţiilor de forma (1). Se demonstrează apoi teorema 2, din care se deduc cîteva cazuri particulare remarcabile.

Seminarul de Matematică şi Fizică
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UBER DAS LOSEN EINER INTEGRALGLEICHUNG AUS DER HYDRODYNAMIK (II).

Kovacs A.

In dieser Arbeit wird das Studium erzüglich der Möglichkeit des Lösens einer Integralgleichung für das geschwindigkeistpotential $\varphi : D \to R^2$, das in der Arbeit [3] angefangen wurde, fortgesetzt.

Lehrsatz 1. [2] . Im Falle der Subsonisch, axialen Bewegung des idealen kompresiblen Fluids durch Profilnetze erfüllt das Geschwingkeitpotential $\varphi : D \to R^2$ folgende Integralgleichung:

$$\varphi(s) + \int_{L_0} \varphi(\sigma) \frac{dM(s,\sigma)}{d\sigma} d\sigma = b(s) + \iint_{D_{0*}} q^*(\sigma) N(s,\sigma)\, d\xi\, d\eta \qquad (1)$$

wo: $\sigma(\xi,\eta)$ und $s(x_0,y_0)$ die krummlinigen Koordonaten des laufenden order des festen Punktes des Basisprofils L_0 ist; D_{0*} ist der äußere Bereich L_0 aus einem Hauptperiodizitätsband;

$$b(s) = 2(x_0 V_{\infty x} + y_0 V_{\infty y}) + \Gamma M(s,\sigma_A) + \int_{L_0} [\psi(s) - \psi(\sigma)] \frac{dN}{d\sigma} d\sigma$$

$$N(s,\sigma) = \frac{1}{\tilde{\pi}} \operatorname{arctg} \frac{\operatorname{th} \frac{\tilde{\pi}}{t}(\eta - y_0)}{\operatorname{tg} \frac{\tilde{\pi}}{t}(\xi - y_0)}, \qquad q^*(s,\sigma) = -\frac{\partial \varphi}{\partial \xi} \frac{\partial \ln \rho}{\partial \xi} - \frac{\partial \varphi}{\partial \eta} \frac{\partial \ln \rho}{\partial \eta} \qquad (2)$$

$$N(s,\sigma) = \frac{1}{\tilde{\pi}} \ln \sqrt{\frac{1}{2}\left[\operatorname{ch} \frac{2\tilde{\pi}}{t}(\eta - y_0) - \cos \frac{2\tilde{\pi}}{t}(\xi - x_0)\right]}$$

Γ = Randkurwe um das Profil L_0, t - Paß des Profilnetzes;
$V_{\infty x}$, $V_{\infty y}$- komponenten der mittleren komplexen Geschwindigkeit,
$\psi(s)$ Stromfunktion.

Satz 1 [3]. Im Falle der Subsonisch axialen Bewegung des kompressiblen idealen Fluids durch Profilnetze bestimmt man die Stromfunktion aus folgender Bedingung bei Grenzwerten

$$\psi(s) = u_0 \cdot \int_0^s p(s)ds, \qquad u_0 = \omega R_0, \quad p = \rho / \rho_0 \qquad (3)$$

ω = Winkelgeschwindingkeit der Rotation des Netzprofils,

R_o = definiert den Ursprung des Bezugssystem bezüglich der Achse der Turbine; ρ und ρ_o sind die Dichten des Fluids in einem irgendeinen Punkt und in dem Punkt mit der Geschwindigkeit Null.

Lehrsatz 2[3]. Im Falle der subsonisch axialen Bewegung des idealen kompresiblen Fluides durch Profilnetze löβt man die integral-diferentialgleichung (1), mit der Methode der Sukzessiven Aproximationen bezüglich der Dichte des Fluides.

Lehrsatz 3. In der ersten Aproximation wird das Lösen der erhaltenen Fredholm Integralgleichung II - ter Ordnung, auf das Lösen von vier Systheme algebrischen Gleichungen zurückgeführt.

Beweiβ . In der ersten Aproximation, wenn man $\rho = \rho_o = konst$ voraussetzt, muss die Integralgleichung Fredholm zweiter Ordnung [3] gelöβt werden

$$\varphi^I(s) + \int_{L_o} \varphi^I(\sigma)\, \frac{dM(s,\sigma)}{d\sigma}\, d\sigma = b(s) \qquad (4)$$

Für das Lösen der Gleichung [4] wird die Regel der Überlagerung der Potenzialströme verwendet und die Lösung sucht man unter der Form:

$$\varphi^I = \varphi_1^I V_{\infty x} + \varphi_2^I V_{\infty y} + \varphi_3^I \Gamma + \varphi_4^I u_o \qquad (5)$$

wo φ_k^I $(k = 1,2,3,4)$ findet man durch das Lösen der Integralgleichung

$$\varphi_k^I(s) + \int_{L_o} \varphi_k^I(\sigma)\, \frac{dM(s,\sigma)}{d\sigma}\, d\sigma = b_k(s), \quad k = 1 \div 4 \qquad (6)$$

wo: $b_1 = 2x_o$, $b_2 = 2y_o$, $b_3 = M(s, \sigma_A)$, $b_4 = \int_{L_o} [\psi^I(s) - \psi^I(\sigma)] \frac{dN}{d\sigma}\, d\sigma$ (7)

Mit der Methode Bogoliubov-Krîlov [1] für die Lösung der Integralgleichung [6], nach der mit einer beliebigen Teilung L_o in n Unterinterwalle $\Delta s = \Delta\sigma$ geteilt wird.

Die Teilung kann nichtuniform sein, z.b. am Angriffbord und am Laufbord wo die Änderung der Funktion φ^I störker ist von Punkt zu Punkt, die Länge der Unterinterwalle kann kleiner sein.

In jedem Unterinterwall wird die Funktion φ^I konstant und gleich φ_j^I vorausgesetzt, wo j die Zahl der Mitle der Konsiderierten Unterinterwalle darstellt.

Wenn in der Lösung der Gleichungen (6) die ersten Punkten der Teilung mit gerade Zahlen notiert werden und die Punkten der Teilung j mit ungerade Zahlen, dann, nach der Methode der Integralgleichung (6) können sie durch folgende Systeme von algebrischen Gleichung aproximiert werden

$$\varphi^{I}_{x_i} + \sum_{j=1}^{2n-1} \varphi^{I}_{k_j} \Delta M_{i,j} = b_{k_i}, \quad k = 1,2,3,4; \; i,j = 1,3,\ldots,(2n-1) \qquad (8)$$

wo

$$b^{I}_{1i} = 2x_i, \quad b^{I}_{2i} = 2y_i, \quad b^{I}_{3i} = M_{i,A}$$

$$b^{I}_{4i} = \sum_{j=1}^{2n-1} \Delta \varphi^{I}_{i,j} \left(\frac{dN}{d\sigma}\right)_{i,j} \Delta\sigma_i, \quad \Delta\varphi^{I}_{i,j} = s_i - \sigma_j, \quad \Delta\sigma_j = \sigma_{j+1} - \sigma_{j-1} \qquad (9)$$

Lehrsatz 4. In der zwiten Aproximation wird das Lösen der erhaltenen Fredholm Integralgleichung auf das Lösen von vier System von Liniaren algebrischen Gleichungen Zurückgeführt.

Beweiß. Wenn man das System (8) löst folgt $\varphi^{I}_{k_i}$ in n verschiedenen Punkten des Umrisses L_o, dann aus (5) folgt φ^{I}_{i} in jedem Punkt des Konturs. Wann man aus

$$\rho^{I} = \rho_o \left(1 - \frac{\gamma-1}{2} \frac{V^{I2}}{c_o}\right)^{1/\gamma-1}, \quad V^{I2} = V^{I2}_{\tau} + V^{I2}_{n}, \quad V^{I}_{\tau} = \frac{d\varphi^{I}}{ds}, \quad V^{I}_{n} = \frac{1}{\rho^{I}} \frac{d\psi^{I}}{ds} \qquad (10)$$

ρ^{I} bestimmt, wird aus (2) ein $q^{*I}(\sigma)$ bestimmt, dann wenn man ernent die Regel der Überlagerung der Potentialströme verwendet, die Lösung der Integralgleichung Fredholm zweiter Ordnung

$$\varphi^{II}(s) + \int_{L_o} \varphi^{II}(\sigma) \frac{dM(s,\sigma)}{d\sigma} d\sigma = b^{II}(s) + \iint_{D_{o\sigma}} q^{*I}(\sigma) N(s,\sigma) d\xi \, d\eta \qquad (11)$$

wird unter folgender Form gesucht:

$$\varphi^{II} = \varphi^{II}_1 V_{\infty x} + \varphi^{II}_2 V_{\infty y} + \varphi^{II}_3 \Gamma + \varphi^{II}_4 u_o \qquad (12)$$

In (12) φ^{II}_k, $k = 1 \div 4$ findet man durch die Lösung der Integralgleichung

$$\varphi^{II}_k(s) + \int_{L_o} \varphi^{II}_k(\sigma) \frac{dM(s,\sigma)}{d\sigma} d\sigma = b^{II}_k(s) \qquad (13)$$

wo:

$$b^{II}_{\ell} = b^{I}_{\ell} + \iint_{D_{o\sigma}} q^{*I}(\sigma) N(s,\sigma) d\xi \, d\eta \qquad \ell = 1,2,3$$

$$b^{II}_4 = \frac{1}{u_o} \int_{L_o} \left[\psi^{II}(s) - \psi^{II}(\sigma)\right] \frac{dN}{d\sigma} + \iint_{D_{o\sigma}} q^{*I}(\sigma) N(s,\sigma) d\xi \, d\eta \qquad (14)$$

$$\psi^{II}(s) = u_o \int_0^s p^{I}(s) ds, \quad u_o = \omega R_o, \quad p^{I} = \rho^{I} / \rho_o$$

Wenn man eine Methode von nummerischen Bestimmungen durchläuft die in Lehrsatz 4 durch die Methode von Bogoliubov-Krîlov gezeigt wurde, folgt das das Lösen jeder Integralgleichung aus (13) auf das Lösen eines Systems von linearen algebrischen Gleichungen zurückgeführt wird.

Für die Lösung der zweifachen Integralle aus (1.) kann die Formel von Simpson für das doppelte Integrall (4) verwendet werden. Die doppelten Integralle sind Nichteigen für $s = \sigma$, aber man kann ihre Konvergenz zeigen. Der Integrationsbereich $D^-_{0\varepsilon}$ wird in die Summe drei Bereiche zerlegt ($D^-_{0\varepsilon} = D^-_0 \cup D^-_1 \cup D^-_2$). Das Bereich D^-_0 enthällt die Kurwe L_0. Im Bereich D^-_0 wird der Diskontinuitätspunkt $s = \sigma$ mit einem kleinem Kreis mit Radius $r = \varepsilon \cdot (d^-_0)$ izoliert.
Natürlich, im Bereich $D^-_0 - d^-_0$ sind die doppelten Integralle konvergent und werden wie in den Bereichen D^-_1 und D^-_2 mit der Formel von Simpson bestimmt.

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ASUPRA REZOLVARII UNEI ECUATII INTEGRALE DIN HIDRODINAMICA (II)

Rezumat

In lucrare s-a arătat posibilitatea rezolvării ecuaţiei integro-diferenţiale a potenţialului vitezelor fluidului compresibil prin reţele de profile prin metoda aproximaţiilor succesive în raport cu densitatea fluidului. In fiecare etapă de iteraţie rezolvarea ecuaţiilor integrale Fredholme de speţa a doua se reduce la rezolvarea unor sisteme de ecuaţii algebrice liniare.

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THE CHARACTERISTIC SPECTRUM ASSOCIATED TO A LINEAR DISCRETE SYSTEM

Topuzu E.

Let X be a Banach space, $A \in \mathcal{L}(X)$ and the discrete Cauchy problem

$$(A;0,x^0) \quad \begin{cases} x_{n+1} = Ax_n \\ x_0 = x^0 \end{cases} \qquad x^0 \in X.$$

with the solution given by $x_n = A^n x_0$, $\forall\, n \geq 0$.

In according with [3], we can associate to the solution $x = (x_n)_{n \geq 0}$ a characteristic number

$$\mu(x) = \mu_A(x^0) = \lim_{n \to \infty} \sup \|A^n x_0\|^{1/n}$$

By analogy with continuous case [1], [2] we call the set of positive real numbers

$$\chi_d(A) = \left\{ \mu_A(x^0) \mid x^0 \in X,\ x^0 \neq 0 \right\}$$

the characteristic discrete spectrum of the operator A, or the characteristic spectrum associated to the linear discrete system (A).

Because $\mu_A(x^0) = \lim_{n \to \infty} \sup \|A^n x_0\|^{1/n} \leq \lim_{n \to \infty} \sup \|A^n\|^{1/n} \|x_0\|^{1/n} = r_A$, where r_A is the spectral radius of the operator A, it results that for any $x^0 \in X$, $\mu_A(x^0) \leq r_A = \max |\sigma(A)|$.

If x^0 is an eigenvector for A, corresponding to the eigenvalue λ, then $A^n x^0 = \lambda^n x^0$ and $\mu_A(x^0) = \lim_{n \to \infty} \sup \|\lambda^n x_0\|^{1/n} = |\lambda|$

We denote $\sigma_p(A)$ the set of all eigenvalues of the operator A. It results that $|\sigma_p(A)| \subset \chi_d(A)$.

Lemma. If $\mathcal{L} \subset s(X)$ is a finite dimensional subspace then there are: a natural number k, $1 \leq k \leq \dim \mathcal{L}$, a finite increasing sequence of numbers $\mu_0 = 0 < \mu_1 < \mu_2 < \ldots < \mu_k \leq \infty$ and a sequence of subspace $\mathcal{L}_0, \mathcal{L}_1, \ldots, \mathcal{L}_k \subset s(X)$ so that

i) $\mathcal{L} = \mathcal{L}_0 \oplus \mathcal{L}_1 \oplus \dots \oplus \mathcal{L}_k$

ii) $\mu(x) = 0$, $\forall\, x \in \mathcal{L}_0$ and $\mu(x) = \mu_i$, $\forall\, x \in \mathcal{L}_i$, $x \neq 0$, $i \neq 0$.

The proof is similar to those of [1].

Proposition 1. If X has finite dimension then $\chi_d(A) = |\sigma(A)|$.

Proof. We suppose that X has dimension m and let $e_1, e_2, \dots, e_m$ be a basis of X. Then the solutions $(A^n e_1)_{n\geq 0}$, $(A^n e_2)_{n\geq 0}, \dots, (A^n e_m)_{n\geq 0}$ represent a basis in the space of all solutions of the system (A).

Let $x^0 \in X$ be, arbitrary. Then $x^0 = \sum_{j=1}^{m} x_j^0 e_j$, so that the solution of Cauchy problem $(A;0,x^0)$ is given by

$$x = (A^n x^0)_{n\geq 0} = \Big(\sum_{j=1}^{m} x_j^0 A^n e_j\Big)_{n\geq 0} = \sum_{j=0}^{m} x_j^0 (A^n e_j)_{n\geq 0}$$

In conclusion, if we denote $\mathcal{S}$ the space of all solution of the system (A) it results that $\dim \mathcal{S} = m$.

By virtue Lemma, $\mathcal{S} = \mathcal{S}_0 \oplus \mathcal{S}_1 \oplus \dots \oplus \mathcal{S}_k$ where $\forall\, x = (x_n)_{n\geq 0} \in \mathcal{S}_j$ it results $\mu(x) = \mu_j$.

Let $\mathfrak{X}_j = \{ x_0 \in X \mid (A^n x_0)_{n\geq 0} \in \mathcal{S}_j \}$, $j = 1,2,\dots,k$.

The subspace $\mathfrak{X}_j$ is invariant to A. It results that $A/\mathfrak{X}_j$ has an eigenvalue λ and an eigenvector x_j^0 and we have

$$\mu_A(x_j^0) = \mu_j = |\lambda| \in |\sigma(A)|, \quad j = 0,1,2,\dots,k$$

hence

$$\chi_d(A) = \{\mu_0, \mu_1, \dots, \mu_k\} \subset |\sigma(A)|$$

On other hand, for every X we have $|\sigma_p(A)| \subset \chi_d(A)$. Because, if X has finite dimension, $\sigma_p(A) = \sigma(A)$, it results $\chi_d(A) = |\sigma(A)|$

Proposition 2. If A has an inverse $A^{-1} \in \mathcal{L}(X)$ then

$$\min |\sigma(A)| = \frac{1}{r_{A^{-1}}} \leq \mu_A(x) \leq r_A = \max |\sigma(A)|, \quad \forall\, x \in X,\ x \neq 0$$

Proof. Indeed, for every $x \in X$, $x \neq 0$, we have

$$\|x\| = \|A^{-n} A^n x\| \leq \|(A^{-1})^n\| \cdot \|A^n x\|$$

therefore

$$\|A^n x\|^{1/n} \geq \frac{1}{\|A^{-1}\|^{1/n}} \|x\|^{1/n}$$

and our assertion results from the properties of spectral radues and the definition of $\mu_A(x)$ by virtue of Proposition 3 from [3]

Proposition 3. For every $A \in \mathcal{L}(X)$ the following inclusion holds: $\chi_d(A) \subset |\sigma(A)|$

Proof. Let suppose that $x_0 \in X$, $x_0 \neq 0$, has the property $\mu_A(x_0) = 0$. Then it is clear that $0 \in \sigma(A)$ (contrary,

$0 < \frac{1}{r_A - 1} \leq \mu_A(x_0) \leq r_A)$ and in this case $\mu_A(x_0) \in |\sigma(A)|$.

Analogously, if $\mu_A(x_0) = r_A$, then $\mu_A(x_0) \in |\sigma(A)|$.

Let suppose now that $0 < \mu_A(x_0) < r_A$. If for every $\lambda \in \sigma(A)$, $\mu_A(x_0) \neq |\lambda|$, then we can denote

$$\sigma_1 = \{\lambda \in \sigma(A) \mid |\lambda| < \mu_A(x_0)\} \qquad \sigma_2 = \{\lambda \in \sigma(A) \mid |\lambda| > \mu_A(x_0)$$

It is easy to see that $\sigma(A) = \sigma_1 \cup \sigma_2$ and σ_1, σ_2 are spectral sets for A. By consequence X has a direct decomposition $X = X_1 \oplus X_2$, X_i, $i = 1,2$ being invariant subspaces of A, and moreover, denoting $A_i = A/X_i$, we have

$$\sigma(A_1) \subset \{\lambda \in \mathbb{C} \mid |\lambda| < \mu_A(x_0)\} \qquad \sigma(A_2) \subset \{\lambda \in \mathbb{C} \mid |\lambda| > \mu_A(x_0)\}$$

Then

$$\mu_A(x) = \mu_{A_1}(x) \leq r_{A_1} < \mu_A(x_0) \quad \text{for } x \in X_1$$

and

$$\mu_A(x) = \mu_{A_2}(x) > \mu_A(x_0) \qquad \text{for } x \in X_2$$

But $x_0 = x_1 + x_2$, $x_1 \in X_1$, $x_2 \in X_2$ and therefore if $x_2 \neq 0$, $\mu_A(x_0) = \max(\mu_A(x_1), \mu_A(x_2)) = \mu_{A_2}(x_2) > \mu_A(x_0)$ and if $x_2 = 0$, $\mu_A(x_0) = \mu_{A_1}(x_0) < \mu_A(x_0)$. The contradiction obtained show as that $\chi_d(A) \subset |\sigma(A)|$.

Remark 1. If we can find $p \geq 1$ such that every solution $x = (x_n)_{n \geq 0}$ of the equation (A) belongs to the space $\ell^p(X)$, then $\chi_d(A) \subset [0,1]$.

Indeed, if $\{A^n x_0\}_{n \geq 0} \in \ell^p(X) \subset \ell^\infty(X)$, then there exists $M_{x_0} > 0$ such that $\|A^n x_0\| \leq M_{x_0}$, $n \geq 0$.

By virtue of uniform boundedness principle we can find $M > 0$ such that $\|A^n\| \leq M$, $n \geq 0$ and therefore

$$0 \leq \mu_A(x_0) \leq r_A = \lim_{n \to \infty} \|A^n\|^{1/n} \leq \lim_{n \to \infty} M^{1/n} = 1$$

The converse is, generally, not true how is showed by following:

Example 1. Let $X = \mathbb{R}^2$ and $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. By virtue Proposition 1, $\chi_d(A) = |\sigma(A)| = \{1\} \subset [0,1]$, while $x_0 = (0,1) \in \mathbb{R}^2$ has the property $\|A^n x_0\| = \|(n,1)\| \to \infty$ and by consequence $(A_n x_0)_{n \geq 0} \notin \ell^\infty \supset \bigcup_{p \geq 1} \ell^p$

Remark 2. If X is an atbitrary Banach space and $A \in \mathcal{L}(X)$, then

$$|\sigma_p(A)| \subset \chi_d(A) \subset |\sigma(A)|$$

Proposition 4. For every $A \in \mathcal{L}(X)$, $r_A \in \chi_d(A)$.

Proof. If $\mu_A(x) < r_A$ for every $x \in X$, then for fixed $x \in X$, we can take $\varepsilon > 0$ such that $\mu_A(x) < r_A - \varepsilon$, i.e. there exists $N_\varepsilon > 0$ with the property $\|A^n x\| \leq N_\varepsilon (r_A - \varepsilon)$, $n \geq 0$.

Since, for $|\lambda| > r_A$, $R(\lambda, A) = \sum_{n=0}^{\infty} \lambda^{-n-1} A^n x$

then

$$\|R(\lambda, A)x\| \leq \frac{N_\varepsilon}{|\lambda|} \sum_{n=0}^{\infty} \frac{(r_A - \varepsilon)^n}{(|\lambda|)^n} \leq \frac{N_\varepsilon}{\varepsilon}$$

and by uniform boundedness principle, there exists $M > 0$ such that $\|R(\lambda, A)\| \leq M$ for every $\lambda \in \mathbb{C}$, $|\lambda| > r_A$, wich is a contradiction ($\|R(\lambda, A)\|$ tends to infinity if $\lambda \in \rho(A)$ tends to the boundary of $\sigma(A)$.

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SPECTRUL CARACTERISTIC ASOCIAT UNUI SISTEM DISCRET

Rezumat

In lucrare se defineşte spectrul caracteristic asociat unui sistem discret şi se stabilesc relaţii cu $|\sigma(A)|$.

Seminarul de Matematică şi Fizică
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ON CONVERGENCE OF CLOSED SETS IN A PROBABILISTIC METRIC SPACE AND PROBABILISTIC DISTANCE FUNCTIONS

Goleţ I.

1. A mapping $F:R \longrightarrow R_+$ is called a distribution function if it is nondecreasing, left-continuous with $\inf F = 0$ and $\sup F = 1$. Let D_+ be the family of all distribution functions F such that $F(0)= 0$. It will be endowed with the natural topology and order. The modified Lévy metric [2] induces the topology of weak convergence on D_+, we shall consider the set D_+ endowed with this topology and we also consider on D_+ the partially order defined by $F \leq G$ if $F(x) \leq G(x)$ for every $x \in R$. Relatively to this order the function H defined by $H_0(x)= 0$ if $x \leq 0$ and $H_0(x)= 1$ if $x > 0$ is the greatest element of D_+.

A Menger space is a ordered triple $(S, \mathcal{F}, T)$ where S is a given set, $\mathcal{F}$ is a mapping of $S \times S$ into D_+ ($\mathcal{F}(p,q)$ will be denoted by F_{pq}) and T is a t-norm, such that the following axioms are verified : 1) $F_{pq} = H_0$ if and only if $p = q$; 2) $F_{pq} = F_{qp}$, for all $p,q \in S$; 3) the triangle inequality

$$F_{pq}(x + y) \geqslant T(F_{pr}(x), F_{rq}(y))$$

for all $p,q,r \in S$ and for all $x \geqslant 0$, $y \geqslant 0$.

It is well known that for every Menger space $(S, \mathcal{F}, T)$ with T continuous, the topology on S induced by the family of (ε,λ)-neighbourhood $\left\{ V_p(\varepsilon,\lambda) = \left\{ q \in S : F_{pq}(\varepsilon) > 1-\lambda, \varepsilon > 0, 0 < \lambda < 1 \right\}, p \in S \right\}$ is Hausdorff.

Let $CL(S)$ denote the nonempty closed subsets of Menger space S. For each $C \in CL(S)$ the probabilistic distance function $F_{\cdot,C} : S \longrightarrow D_+$ is defined by the formula $(F_{\cdot},C)(p)= F_{p,C}$ where

$$F_{p,C}(x) = \sup_{q \in C} F_{p,q}(x).$$

It is easy to see that $F_{p,C} \in D_+$ and $F_{p,C}(x) \geqslant$

$\geqslant T(F_{\cdot\cdot}(x-x_{\cdot})F_{\cdot\cdot}(x_{\cdot}))$ for all $x > 0$, $x_{\cdot} > 0$ and $C \subset S$. We ob-

serve that a point $p \in C$ if and only if $F_{p,C} = H_0$. Let $\{C_n\}$ be a sequence in $CL(S)$ and let $C \in CL(S)$.

Definition. $L_i C_n$ (respectively $L_S C_n$) is the set of points p each neighbourhood of which meets all but finitely (respectively meets infinitely) many sets C_n.

We say $\{C_n\}$ Kuratowski converges [3] to C if $L_i C_n = L_S C_n = C$.

In this note starting from Gerald Beer's ides presented in [1] we examine the relationship between the convergence of a sequence of distance functions $\{F_{\cdot,C_n}\}$ and the Kuratowski convergence of $\{C_n\}$.

2. First, we shall give the following properties of the probabilistic distance function.

Let $(S, \mathcal{F}, \zeta_T)$ be a random normed space over the field Λ [2] then we define $F_{p,C}(x) = \sup_{q \in C} F_{p-q}(x)$.

Proposition 1. If $p,q \in S$, $A,B \subset S$ and $\lambda \in \Lambda$, then we have:

a) $F_{\lambda p, \lambda A}(x) = \lambda \circ F_{p,A}(x)$,

where $(\lambda \circ F)(x) = F(\frac{x}{|\lambda|})$ and $\lambda A = \{\lambda q : q \in A\}$.

b) $F_{p+q,A+B}(x) \geq T(F_{p,A}(x-x_0), F_{q,B}(x_0))$

for all $x > 0$ and $x_0 > 0$.

Proposition 2. Let $(S_1, \mathcal{F}_1, T)$ and $(S_2, \mathcal{F}_2, T)$ be Menger spaces and $(S_1 \times S_2, \mathcal{F}_1 \times \mathcal{F}_2, T)$ the T-product of $(S_1, \mathcal{F}_1, T), (S_2, \mathcal{F}_2, T)$, $(p_1, p_2) \in S_1 \times S_2$, $A = A_1 \times A_2 \subset S_1 \times S_2$, then we have:

$$F_{(p_1,p_2),A}(x) = T(F_{p_1,A_1}(x), F_{p_2,A_2}(x)).$$

Let $(S, \mathcal{F}, T)$ be a Menger space and $C, C_n \in CL(S)$, $n \in N$. We say that $\{F_{\cdot,C_n}\}$ pointwise converges to $F_{\cdot,C}$ if $\{F_{p,C_n}\}$ weak converges to $F_{p,C}$ for all $p \in S$.

Theorem 1. If $\{C_n\}$ is a sequence in $CL(S)$ such that $\{F_{\cdot,C_n}\}$ converges pointwise to $F_{\cdot,C}$ $(C \in CL(S))$, then $\{C_n\}$ Kuratowski converges to C.

Proof. Let $p \in C$ then $F_{p,C} = H_0$ and the weak convergence of $\{F_{p,C_n}\}$ to $F_{p,C}$ implies that for all $\varepsilon > 0$, $\lambda \in (0,1)$ there exists $n(\varepsilon, \lambda) \in N$ such that $V_p(\varepsilon, \lambda) \cap C_n \neq \emptyset$ for all

Seminarul de Matematică şi Fizică
al
Institutului Politehnic "Traian Vuia" Timişoara
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THE SOLVING OF A FUNCTIONAL EQUATION ADD A PROPERTY OF THE HYPERBOLE

Neamţu D., Neamţu N.

D.Pompeiu [1] established the following formula:

$$\frac{x_1 f(x_2) - x_2 f(x_1)}{x_1 - x_2} = f(x_0) - x_0 f'(x_0) \qquad (1)$$

in the hypothesis that f is continue on $[x_1, x_2]$, differentiable on (x_1, x_2) and $x_0 \in (x_1, x_2)$.

I.Stamate [2], [3] considers that the functional equations wich are deduced from (1) substituing x_0 with $\frac{x_1 + x_2}{2}$ or $x_1 x_2$. He solves the equation in the first case, makes some generalizations and in the second case he realizes that one solution in $f(x) = ax^2 + bx + c$ and having the geometric interpretation of formula (1) in view, he gives a property of the parabole.

We propose ourselves to solve the functional equation deduced from (1)

$$\frac{xf(y) - yf(x)}{x - y} = f(\sqrt{xy}) - xy\, f'(\sqrt{xy}) \qquad (2)$$

Noting

$$u(x,y) = \frac{xf(y) - yf(x)}{x - y} \qquad (3)$$

$$v(x,y) = \sqrt{xy}$$

(2) shows us that the functions $u(x,y)$ $v(x,y)$ are dependent, so that

$$\frac{D(u,v)}{D(x,y)} = 0 \qquad (4)$$

Supposing that f is three times differentiable, we obtain $f''(x) = 0$, that means

$$f(x) = ax^2 + bx + c \qquad (5)$$

Proceding in the same way for the functional equation

$$\frac{xf(y) - yf(x)}{x - y} = f\left(\frac{2xy}{x+y}\right) - \frac{2xy}{x+y} f'\left(\frac{2xy}{x+y}\right) \qquad (6)$$

we find the solution

$$f(x) = \frac{a}{x} + bx + c \qquad (7)$$

wich represents in geometry a family of hyperboles. If we have in view the geometric interpretation of formula (1) we are lead to the following property of hyperbole: let be P_1 and P_2 two points on hyperbole (7), S a point on the secant P_1P_2 in the exterior of $[P_1P_2]$, and $M \in [P_1,P_2]$ so that $SM = \frac{2SP_1 \cdot SP_2}{SP_1 + SP_2}$ if T is intersection point of the hyperbole with the parallel through M to OY, than ST is tangent to hyperbole.

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REZOLVAREA UNEI ECUATII FUNCTIONALE SI O PROPRIETATE $\alpha^2 + \alpha i\beta + \alpha i\beta + i\beta^2$

Rezumat

In această notă se rezolvă ecuaţiile funcţionale (2) şi (6) şi se dă o proprietate a hiperbolei.

Seminarul de Matematică şi Fizică
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STUDIUL EFECTULUI DE SERA REALIZAT CU O PELICULA DE ULEI

De Sabata C., Crăciun P., Marcu C.

1. Introducere. Competiţia noilor surse de energie cu sursele clasice este marcată de doi factori, şi anume :

- epuizabilitatea relativă şi absolută a combustibililor clasici faţă de energiile ce se reînoiesc
- costul investiţiilor şi durata de amortizare.

Competivitatea surselor noi - a energiei solare în primul rînd - va fi determinată de reducerea consumului de materiale înglobate şi implicit a energiei înglobate în instalaţii, reducerea preţului de cost a execuţiei, găsirea unor noi procedee de captare şi conversie diferite de cele devenite deja clasice [1].

Intr-o lucrare anterioară [2] am semnalat posibilitatea realizării unor captatoare solare cu cheltuieli minime prin utilizarea unor pelicule de ulei.

Prezenta notă evidenţiază avantajele metodei prin comparaţie cu captatoarele clasice cu efect de seră produs de o placă de sticlă.

2. Măsurători experimentale. Măsurătorile au fost efectuate în condiţii naturale de insolaţie folosind două cuve identice din sticlă şi conţinînd drept agent termic apa. Volumul unei cuve a fost de 8 dm^3 iar suprafaţa de captare de $S_c = 0,053\ m^2$. Pereţii laterali ai cuvelor au fost acoperiţi cu folie reflectantă pentru a se evita pătrunderea radiaţiei solare.

Cuva-martor a funcţionat pe baza efectului de seră realizat cu o placă de sticlă iar cuva de măsură a avut agentul termic acoperit cu o peliculă de ulei (fig.1) şi peliculă de ulei plus placă de sticlă (fig.2).

Grosilea plăcilor de sticlă a fost de 3 mm iar a peliculei de ulei de 3,8 mm. S-a utilizat ulei de parafină cu o bună transp[illegible]

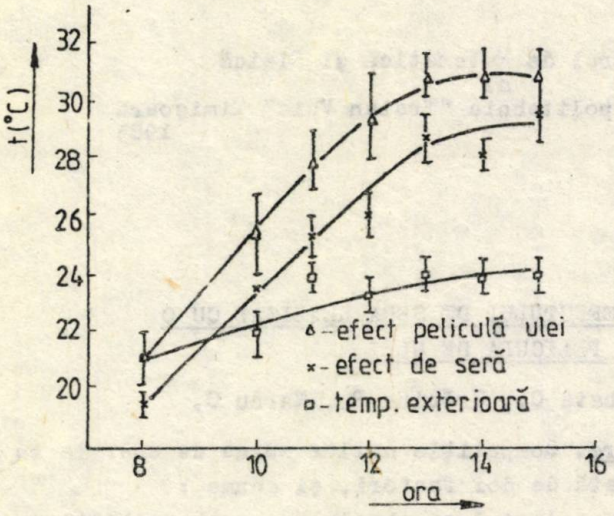

Fig.1

şi o ridicată stabilitate la radiaţia ultravioletă.

Măsurătorile au fost făcute cu cîte un termometru amplasat la mijlocul fiecărei cuve. Rezultatele indicate în grafice au reprezentat media calculată pentru o săptămînă din luna iunie 1985, la diverse ore din zi, ecartul referindu-se la zile diferite şi nu la erori de măsurare.

Determinările experimentale au scos în evidenţă un aspect calitativ important. După încălzirea zilnică a apei din cuve, răcirea nocturnă se făcea diferenţiat, cuva acoperită cu peliculă de ulei răcindu-se mai puţin faţă de cuva-martor acoperită cu placă de sticlă. Drept efect, temperatura de strat s-a modificat treptat ajungîndu-se la o

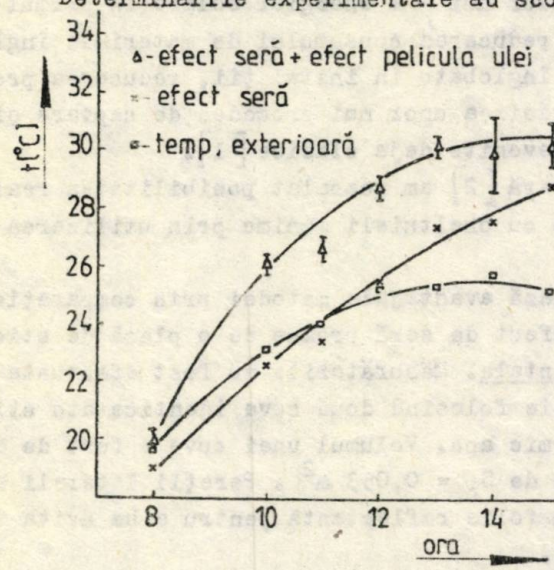

Fig.2

diferenţă de + 2°C în decurs de o săptămînă (fig.1).

Studiul, pe durata mai multor luni, a evaporării apei de sub pelicula de parafină a arătat că prezenţa peliculei elimină practic complet evaporarea.

In fig.3 este redată o curbă de încălzire a apei în cuva aco-

perită cu peliculă de ulei (2) , cu placă de sticlă (1), precum şi insolaţia corespunzătoare zilei, la data de 18 sept.1985. $I\left(\frac{W}{m^2}\right)\times 120$

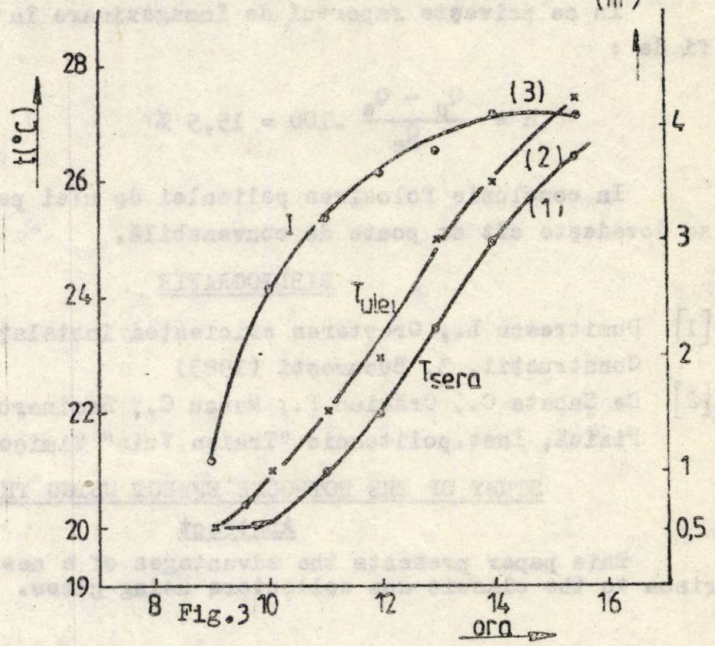

Fig.3

Din curbele de încălzire se observă că pelicula de ulei face ca inerţia termică să fie foarte mică ; placa de sticlă determină o întîrziere cu o oră a începerii încălzirii, după care curbele se menţin paralele.

Curbele din fig.3 permit calculul energiei primite şi a căldurii reţinute în agentul termic al celor 2 cuve.

Energia primită va fi :

$$E = S_i \int_0^t I\,dt = 55.10^4 \text{ J}$$

$S_i = 5,3.10^{-2}$ m^2 ; t = 7 ore.

Căldura înmagazinată va fi în cuva de ulei :

$$Q_u = m.c.\Delta\theta_u^\circ = 34,65.10^4 \text{ J}$$

şi în cuva de sticlă :

$$Q_s = m.c.\Delta\theta_s^\circ = 30,03.10^4 \text{ J}$$

Randamentul captării va fi :

$$\eta_u = \frac{Q_u}{E} = \frac{34,65.10^4}{55.10^4} = 0,63$$

$$\eta_s = \frac{Q_s}{E} = \frac{30,03.10^4}{55.10^4} = 0,54$$

In ce priveşte raportul de înmagazinare în cele 2 cuve el va fi de :

$$R = \frac{Q_u - Q_s}{Q_s} .100 = 15,5\ \%$$

In concluzie folosirea peliculei de ulei pentru efect de seră se dovedeşte cît se poate de convenabilă.

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STUDY OF THE HOTHOUSE EFFECT USING THIN OIL LAYERS

Abstract

This paper presents the advantages of a new method by comperison to the classic sun collectors using glass.

Seminarul de Matematică şi Fizică
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METODA DE MASURARE A ACTIVITATII PULBERILOR SLAB RADIOACTIVE CU CAMERA DE IONIZARE

Crăciun P.

1. Generalităţi

Datorită volumului activ mare care-l are camera de ionizare, aceasta este indicată pentru măsurarea activităţii surselor slab radioactive [1, 2].

Precizia de determinare a activităţii poate fi mărită dacă proba radioactivă se confecţionează sub altă formă decît cea plană.

Dăm în continuare rezultatele obţinute în cazul folosirii unei surse cilindrice în comparaţie cu una plană.

2. Aspectul experimental

Pentru confecţionarea surselor, pulberea radioactivă (azotat de uranil) s-a amestecat cu o soluţie diluată de colofoniu în alcool. Soluţia obţinută se pipetează pe o folie de Al(85x137 mm), care după uscare devine sursă radioactivă. Se poate cunoaşte, din măsurători volumetrice cantitatea de substanţă radioactivă conţinută de sursă.

Sursa astfel confecţionată poate fi plasată în interiorul camerei de ionizare într-o poziţie orizontală sau verticală. Cea de a doua poziţie am realizat-o prin introducerea foliei radioactive într-un cilindru (Ø = 43 mm), cu volum suficient de mic pentru a nu influenţa sensibil capacitatea camerei de ionizare [3] . Sursa devine astfel concentrică cu electrodul colector al camerei de ionizare şi nu introduce erori de volum.

Măsurătorile vitezei de deviaţie (V) a foiţei electrometrului efectuate în prezenţa sursei radioactive montată orizontal cît şi vertical, în interiorul camerei de ionizare, sînt trecute în tabelul următor. Camera de ionizare folosită este de tipul S.G.- 1.

Tabel

Sursa	V(div./min)	$\sigma_I/I = \sigma_V/V$ (%)
Plană	7,0 ± 0,5	7,1
Cilindrică	9,8 ± 0,5	5,1

3.Concluzii

Tabelul de mai sus arată că se poate obţine o îmbunătăţire a preciziei de determinare a curentului de ionizare cu circa 2 % dacă proba radioactivă este confecţionată sub formă cilindrică. Confecţionarea surselor sub această formă este indicată, mai ales, cînd materia analizată este slab radioactivă.

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METHODE DE MESURE DE L'ACTIVITE DES MATERIAUX FAIBLE RADIOACTIVES AVEC LA CHAMBRE DE IONISATION

Résumé

On mésure la radioactivité des échantillons de forme cilindrique. L'éreure de détermination est d'environ 2 %, plus petit que dans le cas des échantillons planes.

Seminarul de Matematică şi Fizică
al
Institutului politehnic "Traian Vuia" Timişoara
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SOME OBSERVATIONS ON MICROPARTICLES

Borneas M.

In the framework of an Integrated-Space-Field Theory [1,2,3] a nonlinear Lagrangien for some microscopic fields has been established, of the form [3]

$$\mathcal{L} = \left[\overleftarrow{\mathcal{H}^{\rho}} V^{\dagger} \partial_{\rho} V + \overrightarrow{\mathcal{H}^{\rho}} (\partial_{\rho} V^{\dagger}) V + (\overleftarrow{\mathcal{H}^{\rho}} - \overrightarrow{\mathcal{H}^{\rho}}) V^{\dagger} i\varepsilon A_{\rho} V + \right.$$

$$\left. + \mathcal{H} V^{\dagger} V \right]_{:} + \mathcal{N} \qquad (1)$$

where V is the microfield, A_{ρ} is the interaction filed, and $\mathcal{N}$ is the nonlinear part ; the notation (:) indicates that in general the bracket must be expressed in (anti-) symmetric form. By dropping the nonlinear part, and by some simple considerations, a Lagrangian has been deduced [2] which leads to the Dirac equation for the electron.

On the other hand a special unitary transformation U has been introduced [4] under which the Lagrangian (1) in interaction is invariant only if V are operators ; but the free Lagrangian is invariant under U even if V are c quantities. This was interpreted that the field V is quantized in particles only in interaction.

A support for the above claim can be found in the work of Aspden [5]. Aspden states that the electron has a natural decay period, and a new electron is immediately created ; and "this creation process is governed physically by interaction with other nearby substance". Some experimental evidence is discussed.

Based on the above considerations and in view of previous results [3], we advance the following assumption. The field V

behaves like wave when free , but gets very high values in a small region - constituting particles - when is in interaction. The size of such a local "swelling" is likely to be

$$r = 2e^2/3mc^2 \qquad (2)$$

according to [5].

It must be noticed that Sapogin [6] sustains similar ideas, considering the particle as a wave packet which periodically appears and disappears.

The view presented in this short note is suitable to contribute to the interpretation of quantum mechanics - a much discussed problem. In fact the wave V is subdued to all undulatory phenomena but we can observe the field only by interacting with it. Hence we observe particles as spots (Shushkin and Fabrikant experiments), or tracks (Wilson chamber), etc. Of course the exact conditions in which the particles appear have to be studied by a comprehensive theory. For the moment let us quote Aspden [7] : "Nature, no doubt, admits a myriad of creation processes, but only shows us the very few which rely upon basic quantum probabilities and determine the existence of the most stable particle forms".

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UNELE OBSERVATII ASUPRA MICROPARTICULELOR

Rezumat

Pe baza unor rezultate anterioare, în concordanţă cu cercetări ale altor autori, se expune idea că microparticulele sînt formaţiuni de mare intensitate ale unui cîmp microscopic, care se formează numai în caz de interacţiune. In fapt observarea particulelor se poate face numai prin interacţiune. Se face o legătură cu interpretarea mecanicii cuantice.

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ON THE DIELECTRIC RESPONSE FUNCTION OF SOLID MATTER IN VARIABLE ELECTRIC FIELD

Cristea Minerva

The experimental data existing in literature concerning the solid dielectrics in variable electric field lead to a more intense study on the dielectric response for solids obeying the Jonscher law [1 - 5]. In this context, the present note establishes the time dependence of the dielectric response function Ψ_{ii}, using the theory of first order perturbations. It is applied to a system with weak coupling and admits the existence of correlated states with infrared-divergent-like response [3].

If one takes the average value of an observable A at the time t [6] :

$$\langle A \rangle_t = \langle A \rangle - \frac{i}{\hbar} \beta \int_{-\infty}^{t} \langle A(t) [H_0, H_{I, \tau - t}(\tau)] \rangle \, d\tau \qquad (1)$$

one observes that it is linear in H_I (the perturbation Hamiltonian in interaction picture), thus it will lead to a linear response.

Let us consider a system with the, polarization operator $\vec{P}$ independent of time and coupling with a variable electric field depending on time $\vec{E}(t)$, such that :

$$H_I = - \vec{P} . \vec{E}(t) \qquad (2)$$

Passing over to the interaction picture, the polarization operator becomes P(t), and

$$\hat{H}_{I, \tau - t}(\tau) = - \hat{\vec{P}}(\tau - t) \vec{E}(\tau) \qquad (3)$$

In view of the time evolution of the operator, the commuta-

tor from (1) can be written :

$$\left[H_o, \hat{H}_{I(\tau-t)}(\tau)\right] = -\left[H_o, \hat{\vec{P}}(\tau-t)\right]\vec{E}(\tau) = -i\,\frac{1}{\hbar}\frac{d\hat{\vec{P}}}{d\tau}(\tau-t).\vec{E}(\tau) \qquad (4)$$

Introducing this result in (1), with A → P, one obtains :

$$\langle P_i \rangle_t = \langle P_i \rangle - \beta \sum_j \int_{-\infty}^{t} \langle \hat{P}_i(t).\, \dot{\hat{P}}_j(\tau-t) \rangle\, E_j(\tau)\, d\tau \qquad (5)$$

where i,j = 1,2,3 and which gives the linear response of the system to a parturbations induced by a variable electric field. Equation (5) can be written in the form :

$$\langle P_i \rangle_t = \langle P_i \rangle + \sum_j \int_{-\infty}^{t} \psi_{ij}(t-\tau)\, E_j(\tau)\, d\tau \qquad (6)$$

where the correlation function of the system has been defined :

$$\psi_{ij}(t-\tau) = -\beta \langle \hat{P}_i(t).\dot{\hat{P}}_j(\tau-t) \rangle \quad ; \quad \beta = 1/k_B T \qquad (7)$$

This function cannot be, in general computed exactly, therefore one must resort to simplified models for its evalution.

In what follows the correlation function is calculated for an isotropic system in which the dipole moment takes on either ~~of~~ of two values $\pm p_o$ on the given direction.

At the time $\tau = 0$, the projection of the polarization vector on the considered direction is p_o. At the time τ, the probability for the projection p_o is $\mathcal{P}_+(\tau)$ and for projection $-p_o$ is $\mathcal{P}_-(\tau)$. Of course the sum is :

$$\mathcal{P}_+(\tau) + \mathcal{P}_-(\tau) = 1 \qquad (8)$$

On the other hand, if one notes with W the transition probability from the state (+) to the state (-) in time unity one obtaines :

$$\frac{d\mathcal{P}_+}{d\tau} = W(\mathcal{P}_- - \mathcal{P}_+) \quad \text{and} \quad \frac{d\mathcal{P}_-}{d\tau} = W(\mathcal{P}_+ - \mathcal{P}_-) \qquad (9)$$

from where : $\frac{d}{d\tau}(\mathcal{P}_+ - \mathcal{P}_-) = 2W(\mathcal{P}_- - \mathcal{P}_+) = 2W(\mathcal{P}_+ - \mathcal{P}_-)$

$$\frac{d(\mathcal{P}_+ - \mathcal{P}_-)}{(\mathcal{P}_+ - \mathcal{P}_-)} = -2W$$

and by integration

$$\mathcal{P}_+(\tau) - \mathcal{P}_-(\tau) = C\, e^{-2W\tau} \quad ; \quad C = \text{integration constant} \qquad (10)$$

If one makes the hypothesis that at the time $\tau = 0$ the system is in the state (+), then it results that $C = \varphi_+(0) = 1$.

From equations (8) and (9) one can write

$$\varphi_+(\tau) = \frac{1}{2}(1 + e^{-2W\tau}) \; ; \quad \varphi_-(\tau) = \frac{1}{2}(1 - e^{-2W\tau}) \qquad (11)$$

These results allow the calculation of the average value $\langle P(0) \cdot P(\tau) \rangle$ where P is the projection of the polarization vector P an the given direction.

By hypothesis $P(0) = p_0$. On the other hand

$$P(\tau) = p_0 \varphi_+(\tau) - p_0 \varphi_-(\tau) = p_0 e^{-2W\tau}$$

and one has :

$$\langle P(0) \cdot P(\tau) \rangle = p_0^2 e^{-2W\tau} \qquad (12)$$

Noting the direction of the projection of the polarization with i (the direction of the electric field), one obtains the expression for the correlation function by the derivation of eq. (12) with respect to :

$$\psi_{ii}(\tau) = 2\beta p_0^2 W e^{-2W\tau} \qquad (13)$$

If we have a system where W is not constant in time, equation (13) can be generalized [3] like this :

$$\psi_{ii}(\tau) = 2\beta p_0^2 W(\tau) e^{-2\int_0^{\tau} W(t)dt} \qquad (14)$$

Acting in this way, the problem of the determination of the correlation function reduces to knowing the transition probabilities in time unity between the two polarization states $(+ p_0)$ and $(- p_0)$.

The function form (14) is similar to the one found by K.L.Ngai and C.T.White [3] .

These results contribute to a better understanding of the solid dielectric behaviour in dependent of time electric field.

These theoretical results are in good agreement data [1-5] and upholds universal mechnism of the dielectric response of solids.

The mechanism is associated with the existence of low energy excitations of the correlated states [3].

These states are due to atom-atom, molecule-molecule, ion-ion electron-electron or impurities interactions.

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ASUPRA FUNCTIEI DE RASPUNS DIELECTRIC A SOLIDELOR IN CIMP ELECTRIC VARIABIL

Rezumat

In lucrare se stabileşte dependenţa de timp a funcţiei de răspuns dielectric în cîmp electric variabil pentru o categorie largă de dielectrici solizi izotropi. Pentru aceasta s-a admis ipoteza stărilor corelate cu răspuns divergent în infraroşu, formulată de K.L.Ngai şi C.T.White şi s-a aplicat teoria perturbaţiilor dependente de timp în aproximaţia de ordinul întîi, unui sistem izotrop cu cuplaj slab.

Seminarul de Matematică şi Fizică
al
Institutului politehnic "Traian Vuia" Timişoara
Noiembrie 1985

INTERPRETATION OF THE BEHAVIOUR OF COUPLED ELECTROLYTIC Ni-Co FILMS USING AN ENERGETIC MODEL

De Sabata C., Marcu C., Gangăl M., Boldan M.

1. Experimental Aspects. This paper is a study regarding the obtaining of coupled electrolytic Ni-Co films deposited on a Cu base in the form of a 20x10x1 mm band ; the superdeposited films had a thickness of 3 μm both for Ni and Co.

1.1. The films have been obtained through successive deposition, first Ni and then Co, from the following baths :

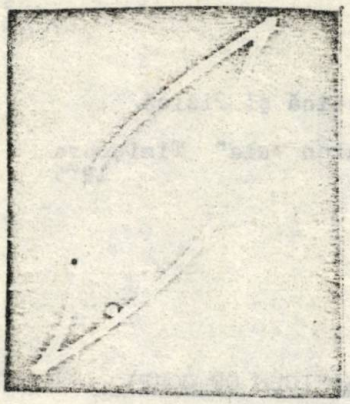
Ni			Co		
$NiSO_4$	8,8	g/l	$CoSO_4$	312,5	g/l
NaCl	12	g/l	NaCl	19,6	g/l
H_3BO_3	15	g/l	H_3BO_3	30	g/l
Saccharin	0,83	g/l	p_H =	3,4	
p_H =	2,7 - 2,6		j =	1 A/dm^2	
j =	3 A/dm^2		θ =	24°C	
θ =	24°C				

The deposited films are adhering, glossy and uniform, of a fine aspect.

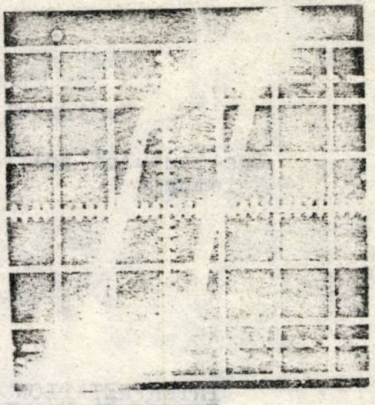
1.2. The magnetic properties have been studied using a set-up that allows a simultaneous increase in the hysteresis loop and in the induced e.m.f. in the process of magnetization, in AC at f = 50 Hz. For the films, the frequency of 50 Hz may be considered as being under quasistatic working conditions [1].

Since the magnetic properties of the films are strongly influenced by impacts or heat treatments, the samples were subject to 3 successive heat treatments, at 150°C , 225°C and 300°C, in air, by linear heating, with a plateau region of an hour and the working and slowcooling temperature in the disconnected furnace

Diegram 1

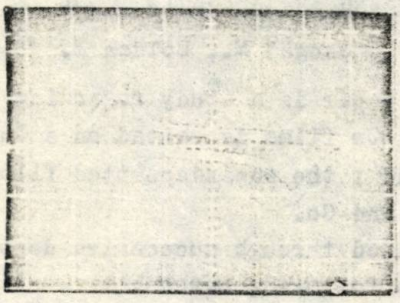


a

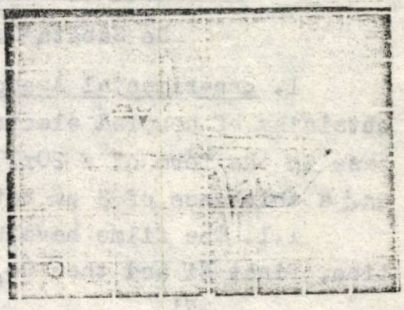


b

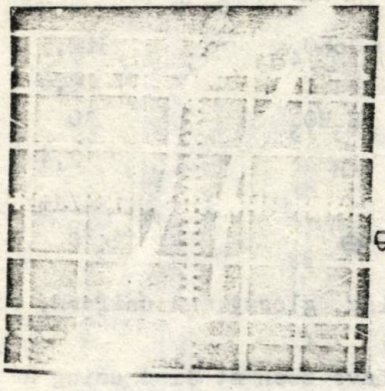
$\theta = 150°C$



c

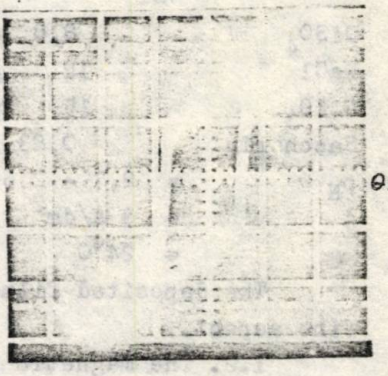


d



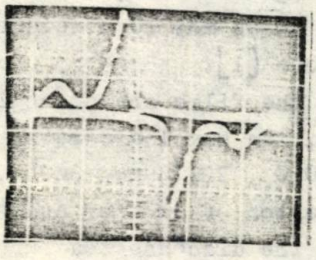
e

$\theta = 225°C$

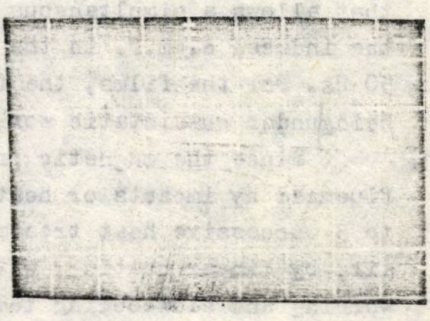


f

$\theta = 300°C$



h



g

Diagram 1 shows the modification in the shape of the hysteresis loop and of the induced e.m.f. as a result of heat treatments.

The coupled NiCo film has a normal "S" loop, the inversion of the magnetic state in an alternating magnetic field occurring over a wide range of values of the coercive field H, a fact observed also on the induced e.m.f. curve. The film behaves as a whole and its hysteresis loop is typical of pure Co films deposited under the same conditions.

Heat treatments bring about essential modifications in the magnetic behaviour of the coupled films; after the second heat treatment (150°C and 225°C), the film shows a loop of the perminvar type and on the induced e.m.f. curve are observed two peaks with clearly separated maxima as the value of the field H, at which they occur. The shape of the impulse obtained in the quasilinear portion indicates the presence of large domains that modify their magnetic state individually by short-time inversions ($\mathcal{E}_i = \frac{d\Phi}{dt}$), the coercive field H_c being reduced to a half as compared to the non-treated sample.

Fig.1

The hysteresis loop and the induced e.m.f. on a Co film.

The second maximum, occurring beyond the opening field H_d, describes a uniform rotation towards the direction of the field H.

The third heat treatment (300°C) changes the aspect of the curves completely, the loop becomes rectangular, the modification of the magnetic state indicating the film's behaviour as a whole, the shape of the hysteresis loop shown in Diagram 1 being similar to that of pure Ni films subjected to the same heat treatments (fig.2).

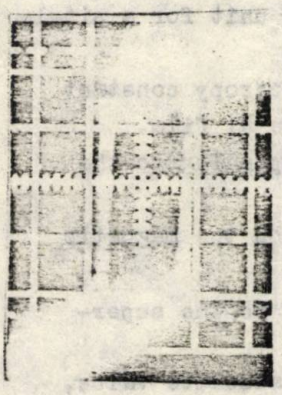


Fig.2

Ni film subjected to e heat treatments, up to 300°C.

As a first observation we can state that at the beginning, the coupled film behaves as if it were made of Co, whereas

after heat treatments, it behaves as if it were made of heat-treated Ni.

2. Theoretical Aspects. The interpretation of the behaviour of coupled NiCo films in a magnetic field and under the influence of heat treatments may be accomplished by using the Stoner-Wohlforth model for a single domaine of spontaneous magnetization [2,3,4].

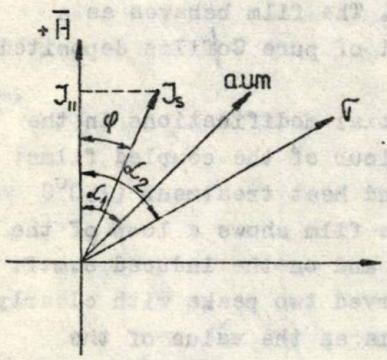


Fig.3

Single domain of spontaneous magnetization. Orientation of main magnitudes.

We have used H for the exterior, coercive magnetic field, J_s for the spontaneous magnetization vector, a.u.m. for the preferential axis spontaneously induced during electrolysis and σ for the mechanical stresses occuring because of the electrolytic coupling of the two films.

Taking account of the symmetry of the sample we assume $\alpha_1 = 45^\circ$, which corresponds to the easy axis of $\langle 111 \rangle$ Ni, and $\alpha_2 = 60^\circ$, which corresponds to the hexagonal structure of Co.

The experimental set-up determines on the oscilloscope screen the pair of values ($J_{||}$, H) where $J_{||} = J_s \cos \varphi$.

We shall write the free energy of a volume unit for a single domain using the relations :

K_u - the uniaxial magnetocrystalline anisotropy constant which describes the properties of the Ni film.

λ_s - the magnetostriction constant corresponding to the boundary between the films.

K_o - the constant that does not depend upon the magnetic properties of the material.

σ - the mechanical stresses occuring due to the superdeposition of Co over Ni.

We shall use the constants taken in their absolute value, The free energy will be written :

$$V = K_o + K_u \sin^2(\alpha_1 - \varphi) + \frac{3}{2} \lambda_s \cdot \sigma \cos^2(\alpha_2 - \varphi) - J_s \cdot H \cos \varphi$$

culate :

$$\frac{\partial Y}{\partial \varphi} = K_u \cdot \sin 2(\alpha_1 - \varphi) + \frac{3}{2} \lambda_s \cdot \sigma \sin 2(\alpha_2 - \varphi) + J_s H \cos \varphi$$

From the consideration mentioned above $\alpha_1 = 45^\circ, \alpha_2 = 60^\circ$ we write :

$$\frac{\partial Y}{\partial \varphi} = K_u \cdot \cos 2\varphi + \frac{3}{2} \lambda_s \cdot \sigma \sin(120 - 2\varphi) + J_s H \sin \varphi = 0$$

which yields :

$$H = - \frac{K_u}{J_s} \cdot \frac{\cos^2 \varphi}{\sin \varphi} + \frac{3}{2} \frac{\lambda_s \sigma}{J_s} \frac{\sin(120 - 2\varphi)}{\sin \varphi}$$

and we shall use the usual notation :

$$\frac{2 K_u}{J_s} = H_K \quad \text{and} \quad \frac{3}{2} \frac{\lambda_s \sigma}{J_s} = H_a$$

This yields :

$$H_1 = - \frac{1}{2} H_K \frac{\cos 2\varphi}{\sin \varphi} + H_a \frac{\sin(120 - 2\varphi)}{\sin \varphi}$$

where : H_K — is the magnetic anisotropy field
H_a — is the field of the mecanical stress in the material.

Observation 1. Field H_1, obtained by nullifying the first derivative of the free energy expression, contains 2 terms whose importance will be different in various possible situations.

The second derivative of the function describing the free energy will be :

$$\frac{\partial^2 Y}{\partial \varphi^2} = - 2K_u \cdot \sin 2\varphi - 2 \frac{3}{2} \lambda_s \sigma \cos(120 - 2\varphi) + J_s \cdot H \cos \varphi = 0$$

and after cancelling it we obtain :

$$H_2 = - H_K \frac{\sin 2\varphi}{\cos \varphi} - 2 H_a \frac{\cos(120 - 2\varphi)}{\cos \varphi}$$

to which we add the magnitude of interest to us :

$$J_{\parallel} = J_s \cos \varphi \qquad \overline{J_s} = \overline{ct}$$

where : $J_{\parallel}$ - being the projection of vector $\overline{J_s}$ on the direc

of the exterior magnetic field $\bar{H}$.

Field H_2 separates the maxima and minima of the free energy fonction. It is also formed of 2 terms as $\bar{H}_1$ but with double values for the constants.

The stable states of vector J_s will be given by :

$$\frac{dY}{d\varphi} = 0 \qquad \frac{d^2Y}{d\varphi^2} > 0$$

and the jump states towards new stable directions, by :

$$\frac{dY}{d\varphi} = 0 \qquad \frac{d^2Y}{d\varphi^2} < 0$$

Observation 2. In the magnetic saturation state in the sens $+\bar{H}$ (fig.3), vector $\bar{J}_s$ will be superposed over field $\bar{H}$. The decrease in its value approaching zero ($\bar{H} \to 0$) will determine the shift of $\bar{J}_s$ from the connection with the field to the connection with the preferential axis (u.a.m.), then with the mechanical stresses ($\bar{\sigma}$), to the inversion of the field $\bar{H}$ and an increase in its value in an opposite sense ($-\bar{H}$), for a certain value, J_s also will inverse its orientation reaching saturation in an opposite sens imposed by $-\bar{H}$.

We shall further study the value of the reduced free energy function Y^* as a function of $\bar{H}$ and φ, the reduced function being obtained by dividing function Y by J_s and neglecting the term K_o which does not depend on the magnetic properties of the material.

Thus :

$$Y^* = \frac{1}{2} H_K \sin^2(45 - \varphi) + H_a \cos^2(60 - \varphi) - H.\cos\varphi$$

Case 1. Non-heat-treated coupled Ni and Co films.

As the oscillograms are obtained in relative values, we shall use the same procedure for the values in Y^* also.

In the case of the normal "S" loop the magnetic anisotropy field and the stress field must be comparable and the relatively high value of the coercive field H_c indicates that the 2 fields are important in terms of value. We shall choose :

$$H_K = 2 \qquad H_a = 2$$

and we shall establish parametres for the coercive filed $\bar{H} \in [2{,}2 \text{ and } -2{,}2]$ and the angle $\varphi \in [0^\circ, 180^\circ]$ with a step of 10°.

Diagram 2

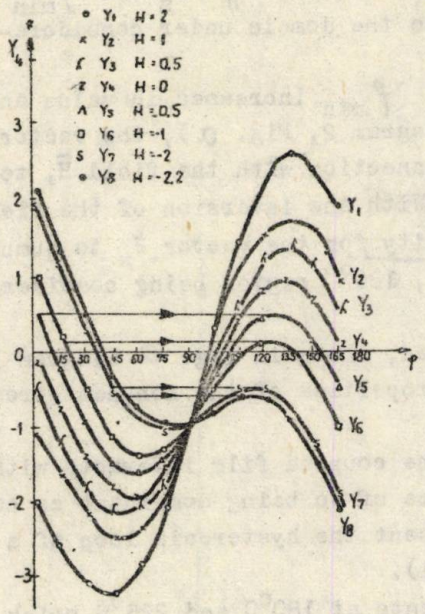


Fig. a

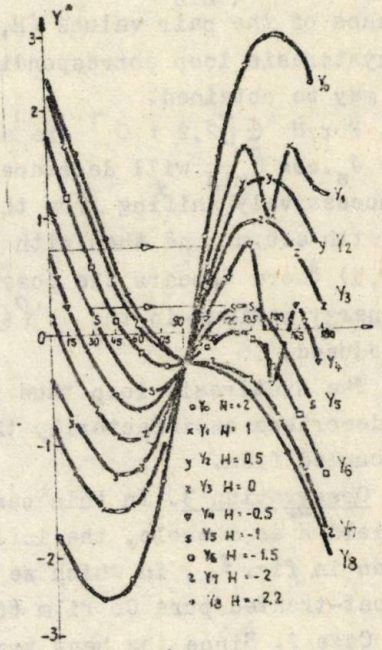


Fig. b

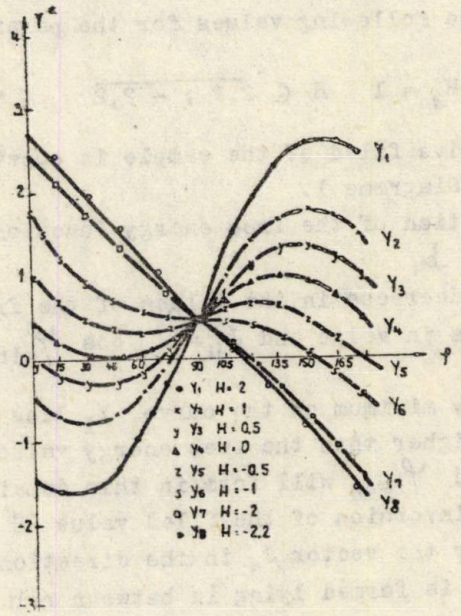


Figure a in Diagram 2 shows $Y^* = f(H, \varphi)$; for each value of $\vec{H}$ an angle φ_{min} is obtained, which minimizes function Y^*. By means of the pair values $(H, J_{\parallel})$ obtained $J_{\parallel} = J_s \cdot \cos \varphi_{min}$ the hysteresis loop corresponding to the domain under consideration may be obtained.

For $H \in [2,2 ; 0]$ the angle φ_{min} increases in value and $J_{\parallel} = J_s \cdot \cos \varphi_{min}$ will decrease (Diagram 2, Fig. a), the vector J_s successively shifing from the connection with the field $\bar{H}$, to that with a.u.m. and then with τ. With the inversion of the field ($H \leftarrow 0,5$) there appears the possibility for the vector J_s to jump, the energy maximum in the $\varphi \in \overline{90^0, 180^0}$ region being considerably reduced.

The hysteresis loop thus formed, shown in fig. a Diagram 3, also describes satisfactorily the properties of the non-heat-treated coupled film.

Observation 3. In this case the coupled film interacts with the field $\bar{H}$ as a whole, the influence of Co being domainant as can be seen in fig. 1 , in which we present the hysteresis loop of a non-heat-treated pure Co film (fig.1).

Case 2. Since the heat treatments at 180°C and 225°C bring about the same effect, we shall analyze only the second case. Since these relaxation heat treatments reduce the stress field by lattice ordering and the migration of the occluded hydrogen, we shall assume the following values for the parametres :

$$H_K = 2 \quad H_a = 1 \quad H \in \overline{2,2 ; -2,2} \qquad \varphi \in \overline{0^0, 180^0}$$

The coercive filed of the sample is considerably reduced, as can be seen in Diagrame 1.

The variation of the free energy function is presented in Diagram 2, fig. b.

With the decrease in the values of the filed $\bar{H} \in [2,2 \rightarrow 0]$, φ_{min} increases in value and $J_{\parallel} = J_s \cos \varphi_{min}$ decreases monotonously.

The energy minimum on the curve Y_1 lies in between 2 maxima that are much higher than the free energy value in the region $\varphi \in \overline{0^0 - 90^0}$ and φ_{min} will form in this domain.

With the inversion of the filed value ($H = -0,5$) there may occur a jump for the vector J_s in the direction $\varphi_{min} = 150^0$ where a minimum is formed lying in between relatively lower maxima.

Diagram 3

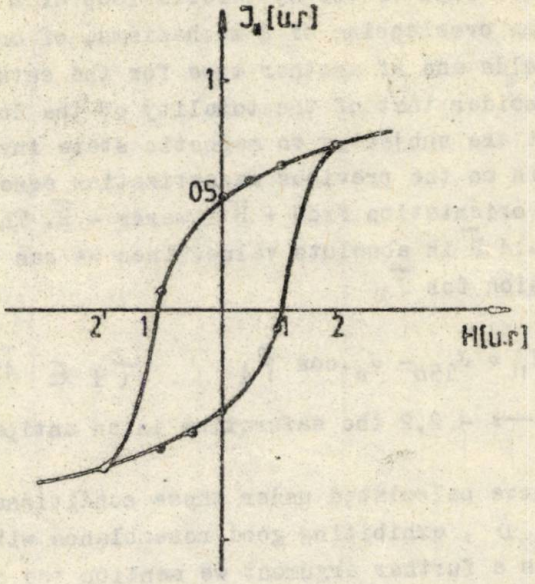


fig. a

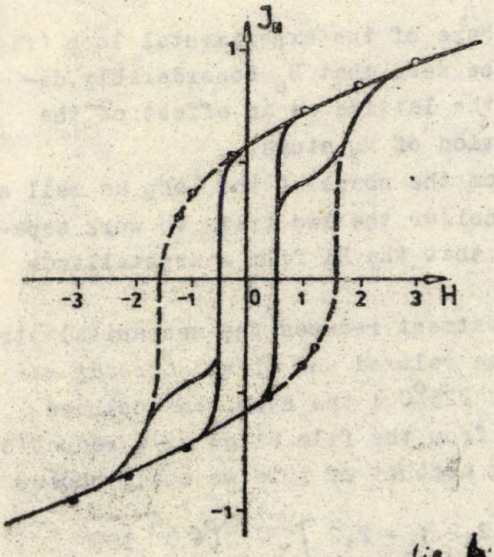


fig. b

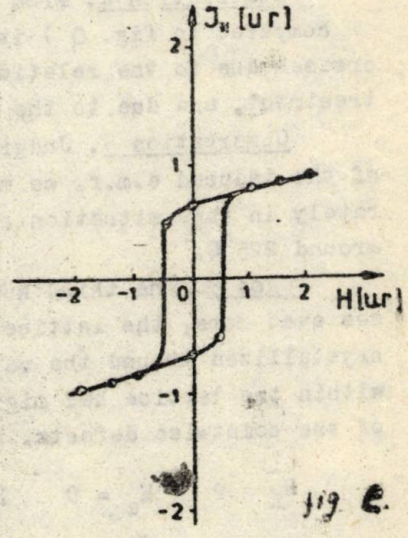


fig c.

The formation of the hysteresis loop of a parminvar type- results in an overlapping of 2 mechanisms, of one type for weaker magnetic fields and of another type for the saturation fields.

We consider that of the totality of the domains forming the film, a part are subjected to magnetic state inversion whereas others remain on the previous magnetization sens, gradually modifying their orientation from + $\overline{H}$ towards - $\overline{H}$. They call for a more intense field $\overline{H}$ in absolute value. Then we can write the following expression for $\overline{J}_{||}$:

$$J_{||} = J_{150} - J_s \cdot \cos \varphi_i \qquad \varphi_i \in \overline{45^\circ, 90^\circ}$$

and, when H $\longrightarrow$ - 2,2 the saturation in an antiparallel sense is obtained.

The curve calculated under these conditions is shown in Diagram 3, Fig. *b* , exhibiting good resemblance with the experimental curve. As a further argument we mention the shape of the induced e.m.f. curve upon inversion of magnetization (Diagram 1, fig. *d*) on which a high peak occurs for low values of H (large domains rapidly modifying their orientation) and a second lower and broader maximum, in intense H filed which show a gradual modification of the magnetic state.

Observation 4. From the shape of the experimental loop (fig. *e* compared to fig. *a*) it can be seen that H_c considerably decreases due to the relation of the lattice as an effect of the treatment, and due to the migration of H_2 atoms.

Observation 5. Judging from the shape of the loop as well as of the induced e.m.f. we may consider the two films to work separately in this situation ; note that the Ni film recrystallizes around 225°C.

Case 3. The third heat treatment reduces the mechanical stresses even more, the lattice became relaxed and Ni had already recrystallized around the value of 225°C ; the hydrogen occluded within the lattice had migrated from the film allowing a reduction of the pointwise defects. Taking account of this we shall choose :

$$H_K = 2 \qquad H_a = 0 \qquad H \in [2{,}2 \; ; \; -2{,}2] \qquad \varphi \in \overline{0^\circ \; 180^\circ}$$

Function Y^* is shown in Diagram 2, fig. *c* . From the pairs of values (H, $J_{||}$) we have calculated the loop shown in Diagram 3, fig. *c* , which is similar to the experimental loop in Diagram 1,

fig. 4.

The inversion of magnetization occurs in intense fields, the film interacts as a whole and the hysteresis loop is similar to that of pure N treated at 300°C (fig.2).

6. Conclusions

This paper has revealed the following aspects :

1. The behaviour of coupled electrolytic Ni-Co films in the process of quasistatic remagnetization may be studied on a single domain model using only three terms which express the connection of the spontaneous magnetization vector $\vec{J}$ with the field $\vec{H}$, with the easy axis and the mechanical stresses σ.

2. Considering the crystal structure of the components, as well as the easy axes defining them, we can admit the following angular values : $\varphi_1 = 85^\circ$, $\varphi_2 = 60^\circ$.

3. The film that is first plated (Ni) forms a matrix for the highly anisotropic particles of the subsequently deposeted film (Co).

4. The magnetic properties of the non-heat-treated material are determined by Co films.

5. After a number of successive heat treatments up to 300°C the film acquires magnetic properties that are similar to the ones of Ni subjected to a similar treatment.

6. Any accidental heating up to temperatures of 100-200°C of the coupled films essentially and irreversibly changes their magnetic properties and implicitly the behaviour of the devise they belong to .

7. Coupled films are advantageous in that due to the interface phenomena, they can be easily adapted to a variety of purposes, their magnetic properties changing over a wide range of values as a result of heat treatments that are easy to perform.

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UTILIZAREA UNUI MODEL ENERGETIC IN INTERPRETAREA COMPORTARII UNOR PELICULE ELECTROLITICE CUPLATE DE Ni-Co

Rezumat

Prezenta lucrare studiază posibilităţile de realizare a unor pelicule electrolitice cuplate de Ni şi Co depuse pe suport de Cu, sub formă de bandă cu dimensiunile de 20x10x1 mm.

S-au studiat proprietăţile magnetice ale peliculelor, urmărindu-se modificarea ciclului de histerezis şi a t.e.m. induse ca urmare a tratamentelor termice (la 150°C, 225°C, 300°C, în aer, cu încălzire liniară, palier de o oră şi temperatura de regim şi răcire lentă în cuptorul deconectat).

Lucrarea cuprinde interpretarea teoretică a comportării peliculei cuplate de Ni-Co în cîmp magnetic şi sub influenţa tratamentelor termice, folosind modelul Stoner-Wohlfarth.

Seminarul de Matematică şi Fizică
al
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RECHERCHES EXPÉRIMENTALES SUR L'EFFICIENCE DE GALVANISATION DE L'ALLIAGE ÉLECTROLYTIQUE NiCoP

De Sabata Coleta, Baeà Rada, Boldan Maria

1. Introduction. La réalisation des dispositifs liés à l'enregistrement, le stockage et la transmission de l'information ont imposé l'étude d'une large gamme de matériaux à base de cobalt, elément dont on apprécie beaucoup l'anisotropie et auquel on associe d'autre éléments non magnetique (Mn,Mg,Zn,P) afin de lui assurer des propriétés mécaniques supérieures, une plus grande dureté et minimiser sa rugosité, respectivement son coefficient de frottement sur l'acier. Le cobalt étant un matériau déficitaire, on a essayé aussi les alliages du nickel et des éléments plus haut mentionnés (Mn,Mg,Zn,P) mais sans obtenir des résultats satisfaisantes [1 ... 5].

Dans l'ouvrage on a étudier un alliage qui puisse à la fois disposer du plus grand nombre possible de propriétés de chaque élément composant pris à part, et d'une efficience élevée dans la réalisation des couches par voie électrolytique.

2. Conditions de galvanisation. A'alliage NiCoP este réalisé de

$NiCl_2.6\ H_2O$	140 g/l	pH = 2	$\theta = 25^oC$
$CoCl_2.6\ H_2O$	140 g/l	$j \in \overline{1,10}$ A/dm^2	
NH_4Cl	90 g/l		
$NaH_2PO_2.H_2O$	10 g/l	dépôt sur feuilles du Cu	

On a étudié le dépôt électrolytique sur une large plage de la densité du courant, car dans les applications technologiques ce sont surtout les grandes densités du courant d'électrolyse qui intéressent, puisque augmentant la vitesse de galvanisation. L'expérience

montre pourtant que, la présence des tensions mécaniques élevées ne se manifeste pas dans ces cas ce qui ne peut pas déterminer l'exfoliation de celle-ci de son suport. En plus, les propriétés magnétiques sont déterminées essentiellement aussi bien par les défauts du réseau cristallin - défauts causés par l'inclusion d'hydrogène dans la pellicule -, que par le champ des tensions mécaniques qui engendrent des directions préférentialles d'aimantation.

L'épaisseur des pellicules étudiées est de d = (4-10) mm. Bien que le produit (j.t) ait été le même pour toutes les couches l'influence de l'efficience η_d s'est manifestée d'une couche à l'autre

L'efficience de l'électrolyse a été étudiés en comparant les couches par l'intermédiaire du facteur η_d défini par :

$$\eta_d = \frac{\Delta m}{j.t} = f(j) \qquad (1)$$

où : Δm - la masse du dépôt [kg] ; j - la densité du courant [A/dm^2] ; t - le temps [s] , puisque, dans ce cas on ne peut pas appliquer la loi de Faraday. En tenant compte du fait que le phosphore donne des sédiments quantitativement réduits, le rendement calculé du bain, pour Ni et Co, a été de 65 %.

L'étude de la dépendance (1) est synthétisée dans la fig.1. Il en résulte une variation quasi-armonique de l'efficience de sédimentation ayant les maxima situés à j = 2,6, 10 A/dm^2 .

Dans tout ces cas les couches ont été adhérentes et il n'y a pas eu d'exfoliations même aux densités de j = 10 A/dm^2, ce qui est très utile du point du vue technologique.

On a étudié, de même, l'influence du temps de galvanisation sur l'efficience η_d à la densité donnée du courant d'électrolyse (fig.2), en constatant l'existence d'un point maximal qu'il faut prendre en compte à la mise au point d'un processus technologique.

3. Analyse de la structure des épreuves . Réalisée au moyen des rayons X, elle a mis en évidence les aspects suivants :

Le dépôt électrolytique est formé d'un alliage de Ni à structure cubique et α Co à structure hexagonale compacte.

La présence du phosphore dans le bain électrolytique détermine la formation sur le support de la combinaison $Cu_3(PO_4)(OH)_3$ avec dépôt de phosphore pur comme partenaire en alliage, ou avec un dépôt en dessous de 1 %, ce n'est pas mis en évidence

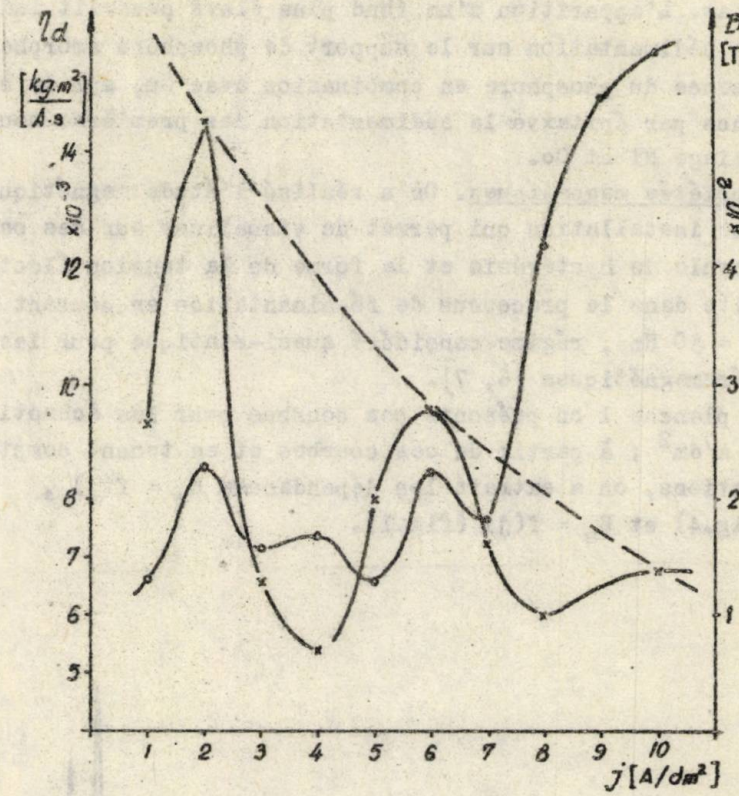
ηd
[kg·m²/A·s]
×10⁻³
B
[T]
×10⁻²
14
12
10
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10
j [A/dm²]

Fig.1

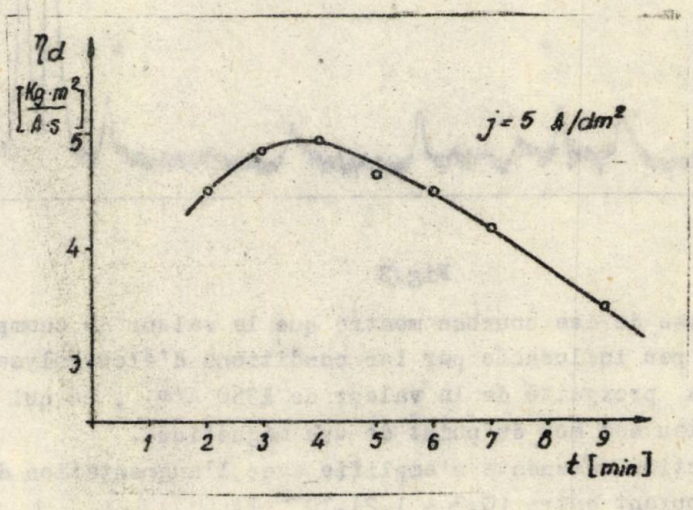
ηd
[Kg·m²/A·s]
5
4
3
j = 5 A/dm²
1 2 3 4 5 6 7 8 9
t [min]

Fig.2

en tant qu'état. L'apparition d'un fond plus élevé pourrait indiquer aussi la sédimentation sur le support de phosphore amorphe.

La présence du phosphore en combinasion avec Cu, sur le support, influence par épitaxye la sédimentation des premières couches de l'alliage Ni et Co.

4. Propriétés magnétiques. On a réalisé l'étude magnétique au moyen d'une installation qui permet de visualiser sur des oscilloscope le cycle de hystérésis et la forme de la tension électromotrice induite dans le processus de ré-aimantation en courant alternatif à f = 50 Hz , régime considéré quasi-statique pour les pellicules féromagnétiques [6, 7].

Dan la planche 1 on présente ces courbes pour les échantillons à $j \in \overline{1,10}$ A/dm^2 ; à partir de ces courbes et en tenant compte des amplifications, on a extrait les dépendances $H_c = f(j)$, $B_r = f(j)$ (fig.4) et $B_s = f(j)$ (fig.1).

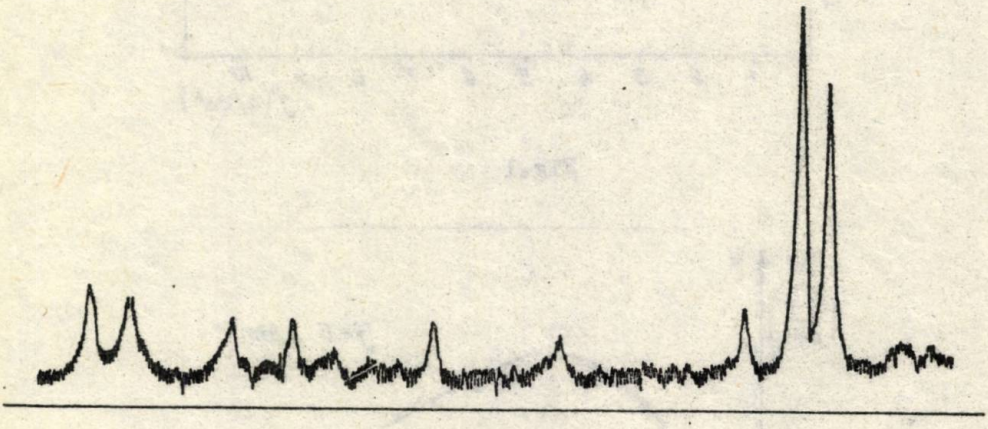

Fig.3

L'analyse de ces courbes montre que la valeur du champ coercitif H_c est peu influencée par les conditions d'électrolyse et se situe dans la proxymité de la valeur de 1360 A/m , ce qui montre que le matériau est mou du point de vue magnétique.

L'induction rémanente s'amplifie avec l'augmentation de la densité du courant entre $(0,3 - 1,2).10^{-2}$ T.

En ce qui concerne l'induction magnétique à la saturation,

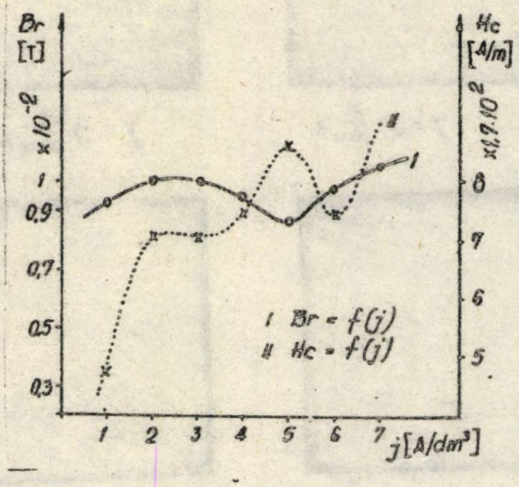

Fig.4

elle présente une variation quasi-armonique en fonction de j, ayant des maxima situés aux mêmes valeurs auxquelles l'efficience de galvanisation est maximale.

Les valuers diminuées du champ coercitif et de l'induction sont dues à la présence du phosphore sur le support [8]; la présence du phosphore dans le bain conduit aussi à l'accoissement de la vitesse de sédimentation et maintient la stabilité du bain.

Les diagrammes de phase de l'alliage triple NiCoP montrent qu=il existe plusieurs types de composants, c'est-à-dire que l'interaction est extrêment variée. On considère, en général, que cet alliage électrolytique peut être une solution solide métastable ou un verre métallique métastable (Electrodepention of alloys, Abner Bremer) , qui ne recristalisent pas aux traitements thermiques, avant 450°C.

Un argument dans le même sens est représenté par la résistance électrique de l'alliage NiCoP qui est de 10-15 fois plus grande que celle des métaux purs qui le composent, Ni ou Co.Ainsi si $\rho_{Ni} = \rho_{Co} = 8\ \mu\Omega$cm, pour NiCoP $\rho_{alliage} = 100\text{-}120\ \mu\Omega$cm [9].

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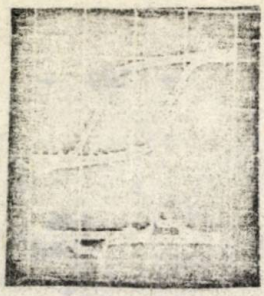
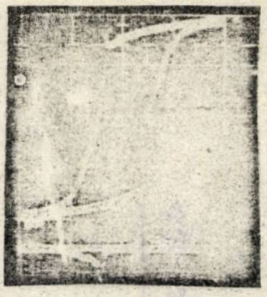
Planșa 1.

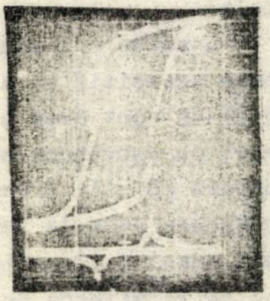
$j = 1 \frac{A}{dm^2}$



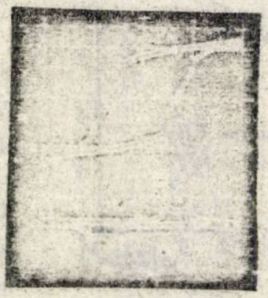
$j = 2 \frac{A}{dm^2}$



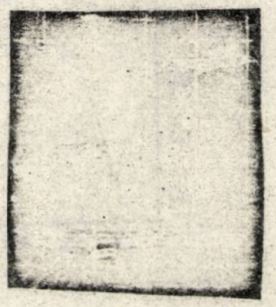
$j = 3 \frac{A}{dm^2}$



$j = 4 \frac{A}{dm^2}$

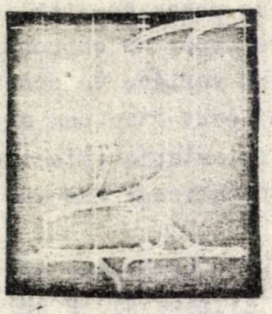


$j = 5 \frac{A}{dm^2}$

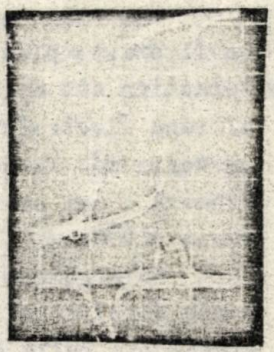


$j = 6 \frac{A}{dm^2}$

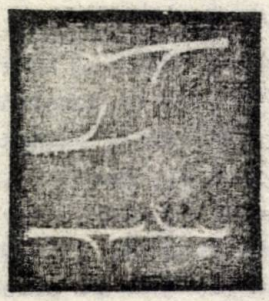
$j = 7 \frac{A}{dm^2}$



$j = 8 \frac{A}{dm^2}$



$j = 9 \frac{A}{dm^2}$



$j = 10 \frac{A}{dm^2}$

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CERCETARI EXPERIMENTALE PRIVIND EFICIENTA DE DEPUNERE A ALIAJULUI ELECTROLITIC NiCoP

Rezumat

In lucrare se prezintă condiţiile de depunere, eficienţa de depunere şi proprietăţile magnetice ale aliajului NiCoP. Aliajul îmbină cît mai multe proprietăţi ale fiecărui component în parte.

Seminarul de Matematică şi Fizică
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Noembrie 1985

ETUDE SUR LA STABILITÉ DES PROPRIÉTÉS MAGNÉTIQUE DE L'ALLIAGE NiCoP AUX TRAITEMENTS ET AUX CHOCS THERMIQUES

De Sabata Coleta, Baea Rada, Boldan Maria

1. Introduction. Les alliages électrolytiques du type NiCoP utilisables sous forme de couches dans l'industrie électrotechnique et électronique, peuvent souffrir des chocs thermiques capables de modifier complètement leurs propriétés magnétiques, ce qui peut conduire à la détérioration de l'ensemble du dispositif qui includ l'élément magnétique [1] .

Afin de mieux connaître cette influence, les couches électrolytiques en NiCoP de [2] ont été soumises à des traitements thermiques successifs à 125°C, 225°C et 300°C, ayant la durée d'une heure et suivis par un refroidissement lent.

On a étudié les modifications concernant la forme et les paramètres des cycles de hystérésis obtenus en courant alternatif à f = 50 Hz, pour les échantillons obtenus dans la plage $j \in \overline{1,7}$ A/dm^2 ; on a étudié aussi la forme de la tension électromotrice induite au cours de la ré-aimantation en courant alternatif, sachant que la forme de cette courbe met en évidence la manière dont intervient la modification de la structure de domaines, respectivement celle des orientations des vecteurs d'aimantation spontanée $\bar{J}_s$ à l'inversion du champ magnétique $\bar{H}$.

2. Résultats expérimentaux. Les planches I et II illustrent l'évolution des courbes dont on parle ci-dessus, en fonction des traitments thermiques.

De ces courbes on à extrait l'évolution de la valeur du champ coercitif qu'on présente dans la fig.1 et l'evolution de la valeur de l'induction de rémanence, présentée dans la fig.2.

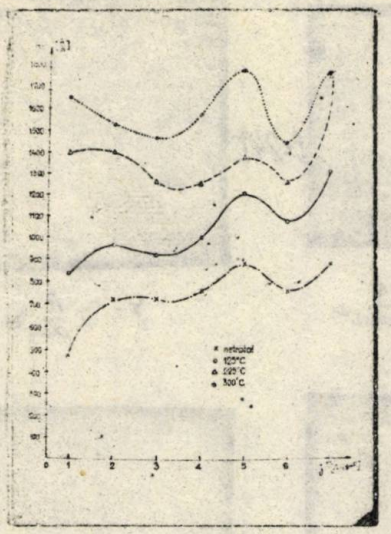


Fig.1

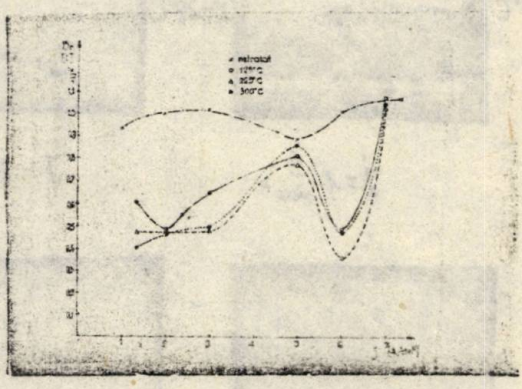


Fig.2

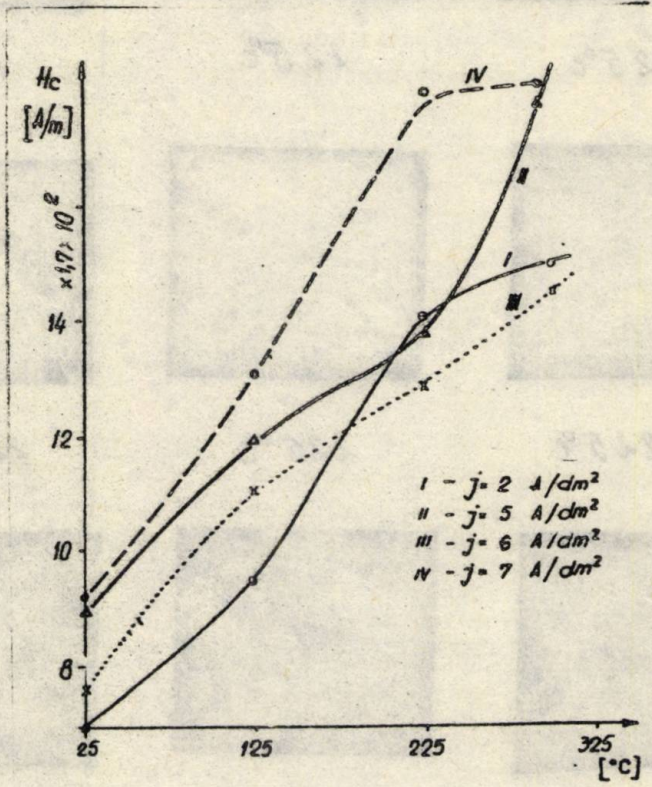
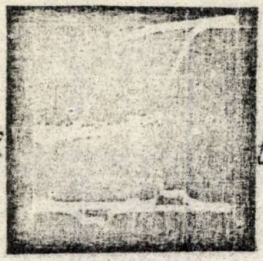


Fig.3

Planche n° 1

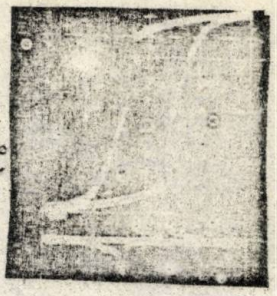
$t = 24°C$

$j = 1 \frac{A}{dm^2}$



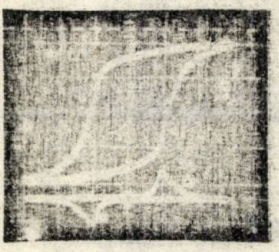
$t = 24°C$

$j = 2 \frac{A}{dm^2}$

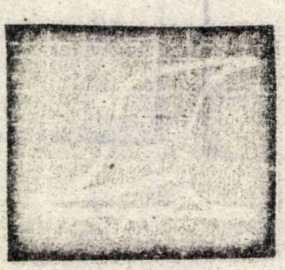


$t = 24°C$

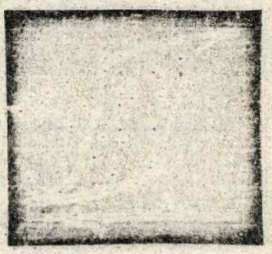
$j = 3 \frac{A}{dm^2}$



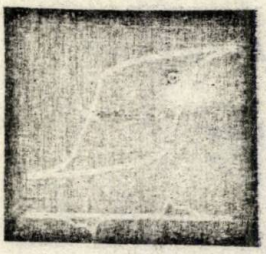
125°C



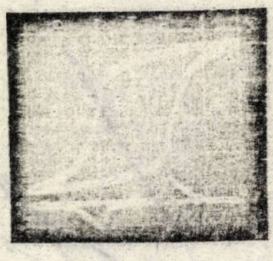
125°C



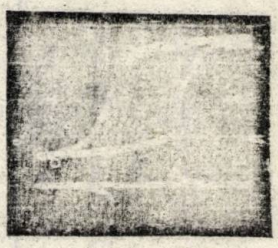
125°C



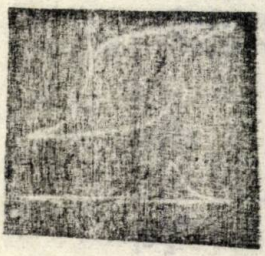
225°C



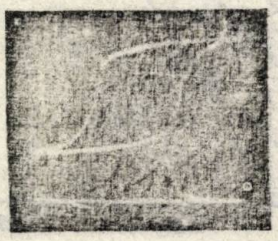
225°C



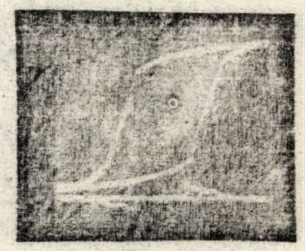
225°C



300°C

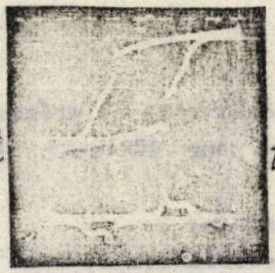


300°C

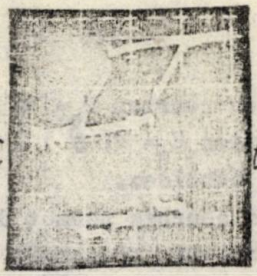


300°C

Planche n°2

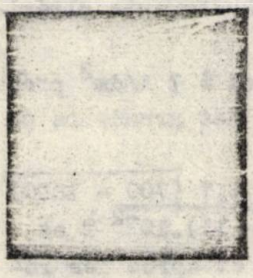


$t = 24°C$ $j = 5 \frac{A}{dm^2}$

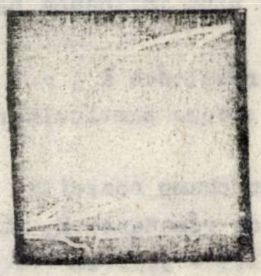


$t = 24°C$ $j = 6 \frac{A}{dm^2}$

$t = 24°C$ $j = 7 \frac{A}{dm^2}$



$t = 125°C$



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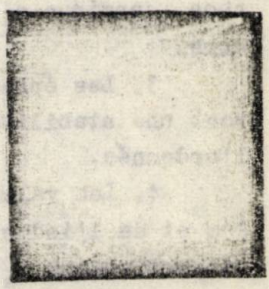

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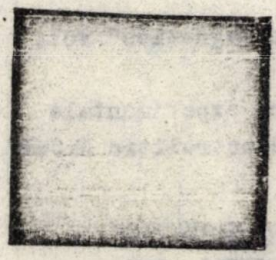

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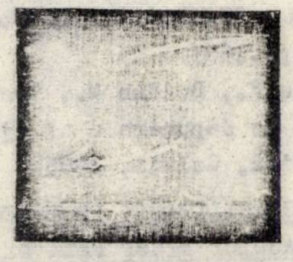
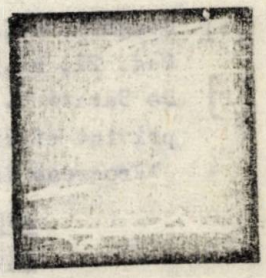


$t = 225°C$



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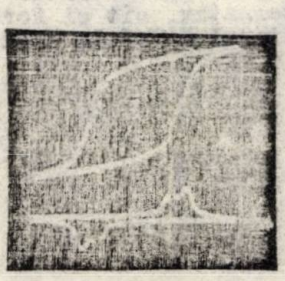
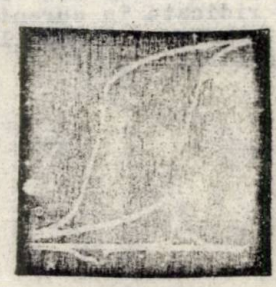
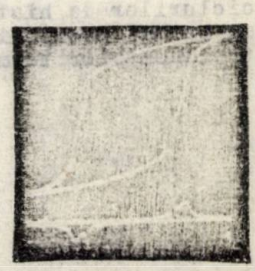


$t = 300°C$



$t = 300°C$



$t = 300°C$

De la famille des courbes $H_c = f(j)$ on a extrait et présenté dans la fig.3 l'évolution $H_c = f(\theta)$ à j = ct pour quelques échantillons réalisées et étudiées.

L'analyse de ces courbes permet de constater :

1. Le champ coercitif s'accroît quasi-linéairement dans l'intervalle de traitement 25 - 300°C, en doublant sa valeur dans tous les cas étudiés.

2. Le champ de l'induction de rémanence diminue au premier choc thermique n'étant plus influencé par réchauffements ultérieurs.

3. Les épreuves résultées à $j = 5$ A/dm^2 et à 7 A/dm^2 présentant une stabilité thermique particulière pour les grandeurs de l'ordonnée.

4. Les valeurs du champ coercitif $H_c \in \overline{1,7\ (700 - 1800)}$ A/m et de l'induction de rémanence $B_r \in (0,3 - 11).10^{-2}$ T et respectivement $B_s \in (1,3 - 5,5).10^{-2}$ T avant et après les réchauffements succesifs mentiennent les pellicules dans le domaine des matériaux magnétiques moux.

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ASUPRA STABILITATII PROPRIETATILOR MAGNETICE ALE ALIAJULUI NiCoP FATA DE TRATAMENTE SAU SOCURI TERMICE

Rezumat

In lucrare s-a urmărit modificarea formei şi a parametrilor ciclurilor de histerezis ridicate în curent alternativ, cît şi forma curbei tensiunii electromotoare induse la aliajul NiCoP în urma tratamentelor termice aplicate.

Seminarul de Matematică şi Fizică
al
Institutului politehnic"Traian Vuia" Timişoara
Noiembrie 1985

MAGNETIC PROPERTIES OF CERTAIN MIXED OXIDES OBTAINED BY WET "FERRITIZATION"

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Brezeanu M. (IPB)

The present paper aims at establishing the magnetic properties of the oxide phases resulting from the decomposition of polynuclear complexes containing iron and nickel, within the reaction medium ; we require this approach due to the strong dependence of the properties in mixed oxides on the way they are produced and thethermal treatment they undergo.

Considering the general preoccupations related to the use of complex polynuclear combinations as generators of mixed oxides,our paper deals with conditions and means of obtaining the nickel ferrite by "wet ferritization", respectvely, by the decomposition of complexes directly within the reaction medium.

Sample achievement. Using the procedure of wet ferritization based on the study of the Fe (II) - Fe(III) - Ni(II) - X-B system, with X being CH_3COOH and B standing for NH_3 presented in [1], we produces mixed exides type $Fe_x^{2+}\ Ni_{1-x}^{2+}\ Fe_2^{3+}\ O_4$, where se has values from 0 to 1, whose composition is controlled mostly by the pH of the reaction medium.

The mixed oxides under investigation were obtained out of polynuclear complexes by wet ferritization without complexing agents [2][3] ; acetic acid was udes as a complexing agent in [1], NH_3 was used for precipitation in both cases.

According to structural estimations [1] the nickel ferrite obtained by the wet procedure results in a maximum rate for pH - 9 - 10, once the reaction medium has been heated for 30-40 hours at 70-90°C.

The resulting samples were thermally treated at 400-1

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according to table 1.

Table 1

Samples lacking the complexing agent	Samples obtained with complexing agent	Temperature of thermal treatment°C (1h)
A 1	B 1	non-calcinet
A 1'(with air barbotage)	-	"
A 2	B 2	400
A 3	B 3	600
A 4	B 4	800
A 5	B 5	1000

Samples of type A 1' were obtained by air barbotage during the precipitation process.

Measuring methods. In dynamic, the cycles of magnetic hysteresis were increased up to frequency of 50 Hz, using an equipment as described in paper [4]. The probing coil of the installation surrounded the pulverous sample sagged inside a glass tube under standard repeatibility conditions.

Experimental results. The experimental installation allowed for a vizualization of the end cycles as prezented in fig.1 (a-f) for the A samples (1-5) and fig.2 (a-f) for the B symples (1-5). The Oy axis involues the magnetization intensity J in relative units (r.u), while Ox the intensity of the field H in the magnetization coil expressed in A/m as follows :

Table 2

Figure	Sample	J r.u/div	H_s [(A/m)/div]	Figure	Sample	J r.u/div	H_s [(A/m)/div]
1a	A1	1	2222	2a	B1	2	2222
1b	A1'	1	2294	2b	B2	2	2345
1c	A2	2	2367	2c	B3	1	2282
1d	A3	2	2303	2d	B4	5	2369
1e	A4	2	2286	2e	B5	5	2286
1f	A5	2	2350				

Table 3, fig.3 and fig.4, presents the influence of the thermal treatment and the complexing agent on the saturation fields H_s , coercive filed H_c and the after-effect factor J_r/J_s .

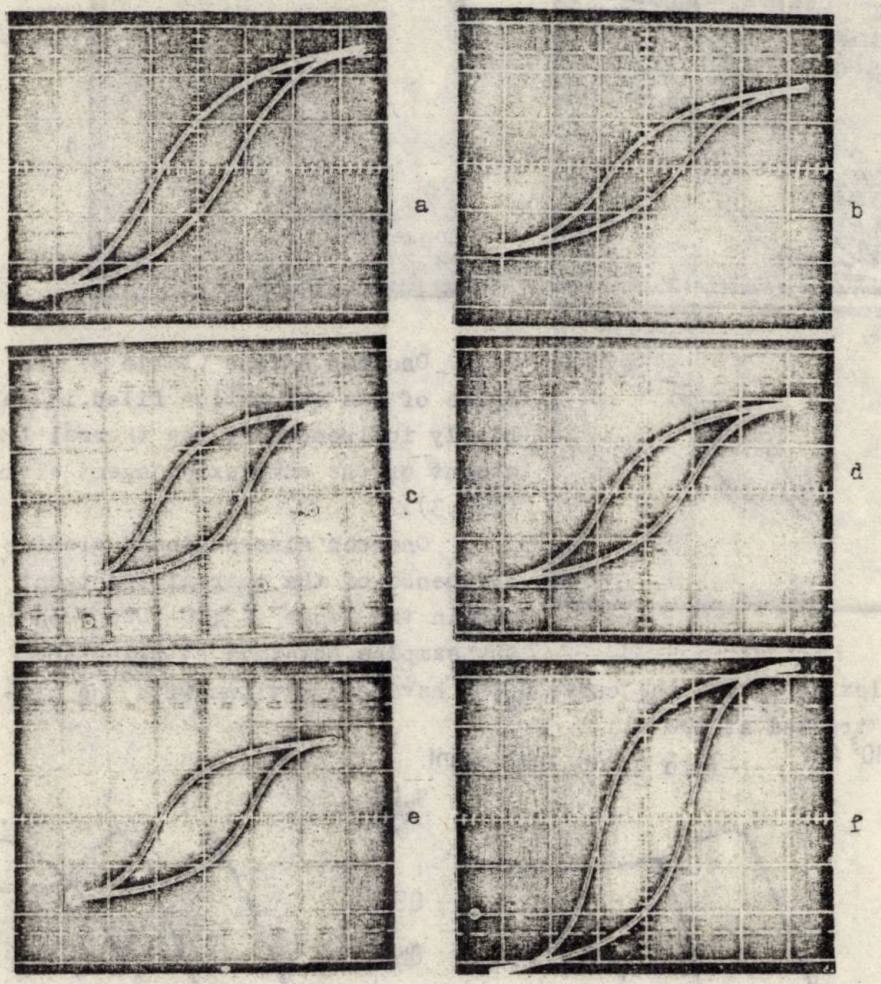


Fig.1

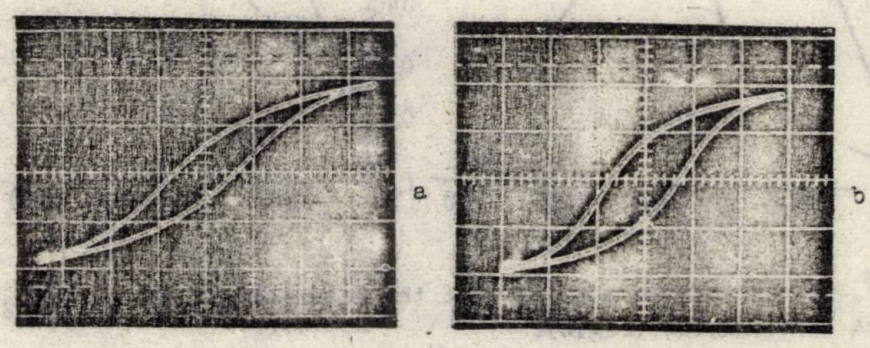
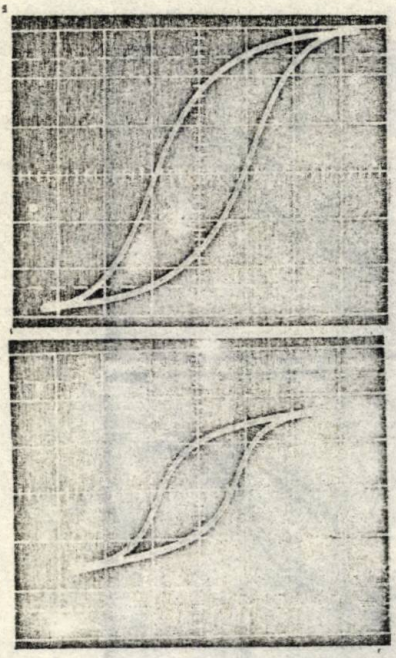


Fig.2



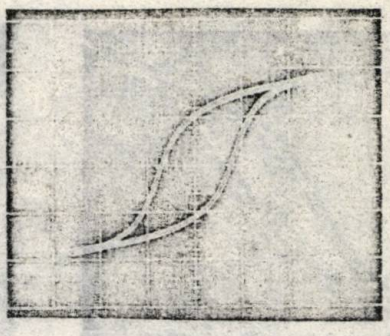


Fig.2

One can notice (table 3) the value of the saturation filed is not highly influenced by the thermal treatment or the complexing agent either (fig.3).

One can also notice a greater influence of the thermal treatment within the tange of 400-1000°C on the samples obtained by using the complexing agent, the curve $H_c(t)$ having a maximum with the samples treated at 600°C.

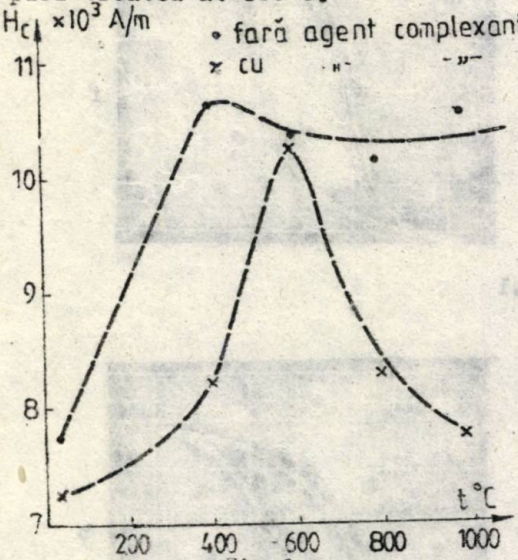


Fig.3

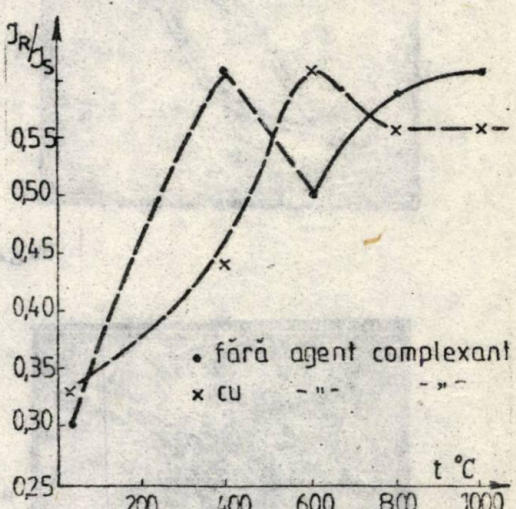


Fig.4

There is also a maximum with the after-effect factor J_R/J_S the same samples (fig.4).

Table 3

Sample	H_S [A/m] x 10^3	Sample	H_S [A/m] x 10^3
A1	39,9	B1	39,9
A1'	38,4	B2	34,0
A2	28,4	B3	38,8
A3	38,0	B4	30,8
A4	32,0	B5	32,0
A5	37,6		

According to their magnetic characteristics these pulverous samples are included in the category of hard magnetic materials having $7,2 \cdot 10^3 A/m \leq H_c \leq 10,6 \cdot 10^3 A/m$.

Conclusions

The estimation of magnetic properties in the oxide phases under investigation adds to the structural information obtained by means of elementary chemical analysis, X - ray diffraction, Mössbauer spectroscopy 1 emphasizing the method suggested by the sama paper for wet ferritization, that may be synthesized as follows :

Complex polynuclear combination $\longrightarrow Fe_3O_4$ + NiO + FP (unchanged previous complex) $\longrightarrow Fe_{3-x}Ni_xO_4$ (solid non-ordered solution) $\rightarrow (Fe^{3+}_xNi_{1-x})[(Fe^{3+},Fe^{2+})_{2-x}Ni_x]O_4 \longrightarrow NiFe_2O_4$ + FP' (solid ordered solution, FP' - superparamagnetic phase) $\xrightarrow{600-1000^oC}$ $NiFe_2O_4$ + α - Fe_2O_3 .

Knowledge of magnetic characteristics contributes to establishing the optimum conditions necessary to obtain nickel ferrite by wet ferritization with the complexing agent CH_3COOH.

A comparison between the above mentioned procedure and the classical methods described in literature results in significant advantages : the raw material to be used does not require special purity conditions and may be obtained by re-using iron waste ; mixing stages and long-lasting measurements are replaced by were precipitation of solutions obtained by dissolving nickel and iron salts in water.

The temperature required 70-90°C and the heating duration 1-48 hours differs from the classical procedure involving 1200°- - 1400°C and 90-100 hours [5] .

The hysteresis cycles confirm that the treatment of oxide samples obtained at 600-1000°C result in two oxide phases, respectively, a clearly shaped nickel ferrite and hematite by the energetical oxidation of incipient magnetite.

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PROPRIETATILE MAGNETICE ALE UNOR OXIZI MICSTI OBTINUTI PRIN PROCEDEUL DE " FERITIZARE " PE CALE UMEDA

Rezumat

In lucrare sînt prezentate ciclurile de histerezis magnetic şi influenţa tratamentului termic asupra caracteristicilor acestora în cazul unor oxizi micşti de nichel şi fier obţinuţi prin descompunerea unor complecşi polinucleari direct în mediul de reacţie.